UNIVERSITY OF LONDON THESIS

Degree PhD Year 2005 Name of Author POVOLEDO, L.

Dynamic Stochastic General Equilibrium Models for The Study of Economic Fluctuations

by

Laura Povoledo

Thesis submitted to University College London for the degree of Doctor of Philosophy

Abstract

The thesis applies a variety of DSGE models to a set of problems whose common link is the analysis of economic fluctuations.

The DSGE methodology is applied first to the analysis of economic fluctuations in Italy. After documenting the crucial features of economic fluctuations in Italy (high volatility of hours worked and low volatility of employment), the thesis explains why the standard RBC model cannot reproduce them. Therefore, a modified RBC model with labour adjustment costs and an underground sector is introduced, and its performance analysed.

Then, the thesis utilizes DSGE theory to study how fluctuations are transmitted within and between countries. Using a two-country general equilibrium model with monopolistic competition and sticky prices, it examines first the relative effects of a wide range (money, supply and demand) of shocks, and then the aggregate effects separately.

The relative effects are the consequences of shocks for the relative price and quantities of domestic tradeables versus nontradeables. The main finding is that not only sector-specific shocks affect these relative prices and allocations, but also aggregate monetary shocks, thus contributing to explain why money has sectoral effects, as in the empirical literature.

The aggregate effects are the consequences of shocks for the main macroeconomic variables. The analysis of the aggregate effects differs from the most recent literature because: 1) the role of critical parameters in the transmission is analysed simultaneously; 2) the analysis is not confined to monetary shocks; 3) supply and demand shocks are disaggregated by sector; 4) the assumptions that the marginal productivity of labour may be decreasing, and that individuals cannot work in both sectors, are introduced.

The aggregate effects of the shocks depend on the choice of parameters. The assumption that individuals cannot work in both sectors leads to a lower elasticity of marginal costs with respect to output.

Acknowledgements

I owe the greatest debt to my supervisors and teachers Wendy Carlin and Gianluca Violante. My thoughts and my attainments would have been lesser without their teachings, their counter-arguments and their example.

I also would like to express my gratitude to all the people who listened to me and offered their advice, especially Vincent Labhard and Nicola Pavoni, to the Bank of England's International Economic Analysis Division for opening its doors to me, and to the ESRC for financial support.

Finally, to my family for bearing with my choices.

Contents

List of Tables

Chapter 1

Introduction

Dynamic Stochastic General Equilibrium (DSGE) models have now become a standard tool for macroeconomic analysis and macroeconomic policy evaluation. Because they fully specify a dynamic optimisation problem for agents under uncertainty, they are characterised by analytical rigour, but at the same time they have proven to be very flexible, as they are able to accommodate a wide class of macroeconomic problems, for example business cycles, monetary and fiscal policy, welfare analysis, wealth distribution and inequality.

As a tool, the DSGE methodology can or should be judged by its ability to give answers, increase understanding or generate a new perspective or viewpoint. Since the seminal works of Kydland and E.C. Prescott (1982), and Long and Plosser (1983), the literature that uses DSGE models has expanded enormously, therefore it is not possible to provide here a comprehensive appraisal based on all the problems that have been dealt using the DSGE methodology. However, it is possible to reach perhaps a tentative, yet deeper understanding by applying this method to a smaller set of problems, which is what this thesis sets out to do.

In the forthcoming chapters, a variety of DSGE models are introduced and applied to a set of problems, all of them related to the analysis of economic fluctuations. Some of the models are used to study real fluctuations, others to study nominal fluctuations; some models are for closed economies and others are for open economies; some models are standard and others possess innovative features. There is some variety also in the set of questions that are analysed in the thesis, providing an overall perspective on the method. In fact, Chapters 2 and 3 are applications of the DSGE methodology to a special case (the analysis of economic fluctuations in Italy), and Chapters 4 and 5 use the DSGE methodology to study how fluctuations are transmitted within and between countries. In particular, Chapter 2 documents the fluctuations occurring in the Italian economy, and then tries to quantify how much they can be explained by standard RBC theory. Chapter 3 presents a DSGE model with labour adjustment costs and the underground sector, in order to explain why in Italy hours of work fluctuate more than employment. Chapter 4 investigates how shocks are transmitted to relative quantities and relative prices within a country. Chapter 5 investigates the domestic and international transmission of shocks to the main macroeconomic variables. Finally, the Conclusion chapter puts forward a "specific" or "particular" appraisal of DSGE models, specifically limited to the analysis of economic fluctuations, and from the particular viewpoint of the set of questions that are analysed in the thesis.

The DSGE methodology is the fruit of the intellectual effort of several scholars. By looking at the "landmark" or key papers, the next Section illustrates how the method came gradually into being, and the advances made at each step.

The first contribution that has proved to be fundamental for the development of DSGE modelling was made by Muth in 1961, but the first fully-formed DSGE models appeared only in the early eighties, with the seminal works of Kydland and Prescott (1982), and Long and Plosser (1983), who introduced the first Real Business Cycle (RBC) models. In between these dates, a clearly-identifiable set of papers provided the conceptualizations and the techniques that are now routinely used in DSGE modelling. These conceptualizations and techniques, rather than the specific theories that have been put forward with them, are the focus of this introductory chapter. In fact, while the first DSGE or RBC models were neoclassical in nature, the recent literature has accommodated a variety of theories or explanations of business cycle fluctuations, proving that the DSGE methodology can be dissociated from particular theories or schools of thoughts.

1.1 The foundations of the DSGE methodology

Muth (1961): Rational Expectations and the Theory of Price Movements

"Rational Expectations and the Theory of Price Movements" by John Muth was not the first paper that introduced a notion of rationality, but it is important because it contains a notion of rationality for the agents' expectations. Rationality is often assumed, for example, in modelling consumer's behaviour: if a preference relation is both transitive and complete, then it is a rational preference relation. Muth was the first to introduce a notion of rationality for the expectations of the agents, and he also showed how this concept can be applied to dynamic models.

Consider, as a simple setting, an isolated market, with a single commodity that cannot be stored, and with the assumption that production lags behind supply. The market equations are:

$$C_t = -\beta p_t \ ,$$

$$P_t = \gamma p_t^e + u_t \ ,$$

$$P_t = C_t \ ,$$

where P_t is the number of units produced, C_t is the amount consumed, p_t is the market price, p_t^e is the expected market price on the basis of the information available up to period $t-1$, and u_t is an error term. The first equation represents demand, the second equation supply, and the last equation represents the market equilibrium.

Solving for the equilibrium price gives us the equation:

$$p_t = -\frac{\gamma}{\beta} p_t^e - \frac{1}{\beta} u_t \ .$$

Then, if the errors have no serial correlation and $Eu_t = 0$, the model's prediction for the price is:

$$Ep_t = -\frac{\gamma}{\beta} p_t^e \ .$$

If the prediction of the theory was different from the expectations of the firms, then there would be opportunities to profit from this knowledge, by inventory speculation, or by operating a firm, or by selling a price forecasting service to a firm. Assuming rationality means that the expectations of the firms are the same as the prediction of the theory, that is:

$$p_t^e = Ep_t \ .$$

Then, if $\gamma / \beta \neq -1$ the rationality assumption implies that $p_t^e = 0$, or that the expected price is equal to the equilibrium price. It is important to point out that the equalisation of the expected price with the equilibrium price is not a necessary implication of the assumption of rational expectations. In this particular case, this result is due to the shocks being completely unpredictable, i.e. $Eu_t = 0$. For example, if the shock could be predicted on the basis of prior information, then:

$$p_t^e = -\frac{1}{\beta + \gamma} Eu_t \ .$$

How desirable is the assumption of rational expectations? The advantages of rational expectations, at least from a purely theoretical point of view, are many. First, the notion is meant to be applied to all the different markets and systems, ensuring consistency. Secondly, if expectations were not rational, there would be opportunities for economists (the people who know the model) in commodity speculation, running a firm, or selling the information. And thirdly, rationality is an assumption that can be modified. Systematic biases, incomplete or incorrect information, poor memory, etc., can be examined with the same analytical methods based on rationality.

Lucas and Rapping (1969): Real Wages, Employment and Inflation

The first piece of research by Lucas into the territory of business cycles is "Real Wages, Employment and Inflation", written in collaboration with Leonard Rapping. The main purpose of the paper is to provide an account of the behaviour of U.S. wages and employment that could be directly brought to the data and estimated.

The model that Lucas and Rapping put forward possessed several innovative features. They wanted a model that reconciled the assumptions that the labour supply does not respond to real-wage changes in the long run (as is normally the case in growth theory), but it is infinitely elastic (as in a Keynesian theory with rigid wages), or positively-sloped, in the short run.

As Lucas and Rapping's model was meant to provide an account of both the long and the short-run elasticities, it could not be a static model. It was an innovation then to introduce two periods in the model, and to assume that households do not simply choose between consumption and leisure in the *current* period, but also between those and consumption and leisure in the *future.* For example, a worker that expects an increase in the future real wage may find it advantageous to increase leisure (work less hours) today, while at the same time increasing today's consumption by dissaving. On the other hand, expected future increases in prices reduce the attractiveness of working today for the purpose of consuming tomorrow, because in this case the real interest rate falls, therefore the returns of saving today for tomorrow are reduced.

With their model, Lucas and Rapping also show that it is possible to conceive a role for money in the labour market. If the household expectations about future real wages and prices are adaptive, as illustrated by the following equations:

$$\ln w_t^* = \lambda \ln w_t + (1-\lambda) \ln w_{t-1}^* + \lambda' \ ,$$

$$\ln P_t^* = \lambda \ln P_t + (1-\lambda) \ln P_{t-1}^* + \lambda'' \ ,$$

where w_t^* and P_t^* are expected future real wages and prices, w_t and P_t are current real wages and prices, and λ' and λ'' are trend terms, then labour supply increases after an increase in money and prices, that is, there is "money illusion". This happens because workers with adaptive expectations expect that the economy will return to higher wages and prices, therefore it is optimal for them to increase their current supply of labour.

The result of money illusion also hinges on the fact that increases in money are followed by falling expected real interest rates, as nominal interest rates fail to adjust fully to the increase in prices. Lower interest rates reduce the attractiveness of working today and saving in order to work less tomorrow. Lucas and Rapping refer to the fall in real interest rates simply as an empirical fact, which they leave unexplained, as they work at the partial equilibrium level. However, this unexplained issue, plus the thought of modelling expectations "consistently" (expectations in one sector of the economy must be compatible with those in another sector), provided the incentive for casting the issue of the effects of money in general-equilibrium terms, and lead to the 1972 paper.

Lucas and Rapping estimate their model with US data and find elasticities for the short and the long-run labour supply that are consistent with the theories, but in addition to this result there are some other extras in their model. The parameters λ' and λ'' are expected trends, which the agents revise whenever they are given "sufficient cause". The government cannot rely on stable private-sector forecasts λ' and λ'', and therefore it has no possibility of raising output indefinitely with a policy of systematic inflation.

Lucas and Rapping's is not a general equilibrium model, it is not stochastic and the dynamics in it is limited to two periods, so at first glance, no matter his noticeable merits, it seems out of place in this survey of DSGE models. However, it is important to start this survey with it because it influenced Lucas' thinking for years afterwards[1]. In fact, it is easy to see that the ideas that intertemporal maximization matters for labour supply, and that agents adapt their decision rules to changes in policy, are already there.

But even more importantly, in this model all labour supply changes happen in equilibrium, a view that it is central in the DSGE literature started later in the 1980s. According to Lucas (1981), working hours actually supplied to the market are what individuals choose. Social conventions and institutions may affect individual decisions, but Lucas thought that institutions cannot ultimately serve other purposes than to aid the matching of preferences with opportunities.

However, it is possible to argue that at least some of the features in the actual labour market prevent households, as a matter of fact, from adjusting optimally their labour supply. This is the case, for example, of fixed working hours, or set contractual arrangements, tying both employers and employees. Nevertheless, the most rigorous, "scientific", approach would be to construct models where these features are themselves explained, as institutions can be chosen or changed, instead of simply delivering models with specific institutional features in the set of assumption. Such models might be too complex, and even possibly out of reach with the tools available now. But the strategy of adding *ad hoc* institutional features to explain behaviour does not seem to be in line with Lucas' approach to macroeconomic modelling.

Lucas (1972): Expectations and the Neutrality of Money

It is difficult to underestimate the influence exerted by "Expectations and the Neutrality of Money", as it contains all the foundation-stones of the DSGE methodology, and innovations which now are routinely used by economists in their models.

The core message of this paper is that money can have an effect on output, even when all prices are market-clearing and agents behave optimally. The only assumption that is

[1] As recognised by Lucas (1995) and (1981).

necessary to obtain this result is that traders only know the price in the market in which they operate, but they do not have information on the aggregate price level. In this way, the real effects of money can be explained without resorting to any device that assumes that agents do not do what they are capable of (adjusting prices), or what they want (behave optimally).

It is possible to argue that the method of solving an economic model by finding the (possibly unique) equilibrium solution is probably as old as economics itself. It requires of course a definition of equilibrium, which can be defined as a situation where agents do not have an incentive to change their behaviour, and all markets clear[2]. In many models money can only affect prices but not allocations, so being able to imagine a situation where money can have real effects because of imperfect information but without any compromise or departure from the equilibrium method is certainly a big step, but there is even more in Lucas' paper.

If somebody asked why a solution to an economic model must be an "equilibrium" (after all, solutions which are not at the equilibrium defined above can also be hypothesized), the best answer is that this is a requirement for consistency. Individuals can always correct their behaviour and formulate their choices optimally, markets clear if they are free to operate.

But Lucas' contribution to this line of thought is an additional requirement for consistency, that is, the requirement that agents use the correct conditional distribution of future variables when calculating their expectations. This "rational expectations" hypothesis is derived from Muth (1961), but now let us see how Lucas made it operational in his model.

At each point in time, there are two generations in the model, the young and the old[3]. Each individual lives for two periods only, and there is no population growth. Output is a function of labour input only. In each period the young must decide their labour supply and therefore output, and their level of money holdings. Output cannot be stored, but money holdings can be carried to the next period in order to buy output for future consumption. That is, in each period the young and the old generations exchange output for money, but trade can only occur in two physically separate markets, to which individuals are allocated. Uncertainty arises because the individuals' allocation is in part

[2]This method thus excludes looking for "disequilibrium" solutions, for example situations with unemployment.

[3]Many essentials of the model are due to Samuelson (1958).

stochastic, and because the money supply is subject to shocks.

The decision problem faced by a member of the young generation is then:

$$\max_{c,n,\lambda}\left\{U(c,n)+\int V\left(\frac{x'\lambda}{p'}\right)dF\left(x',p'\,|m,p\right)\right\} ,$$

subject to:

$$p(n-c)-\lambda\geq 0 .$$

where c is consumption, n is labour supply, λ are nominal balances, augmented by next period's transfers x', p is the price level, and m is money supply. In the above equation, current period variables are related to the future unknown variables x' and p'. Because of the uncertainty in the economy, individuals need to take the expectation of V, using the conditional distribution F. Expectations are formed rationally because the evaluation of F is part of the solution to the problem above, or alternatively F must be consistent with the solution. When solving the model individuals are able to write prices as a function of all possible states of the economy. Since the distribution of future stochastic shocks is known to them, they are able to calculate the correct probability of the future p', conditional on state of the economy today.

The assumption that agents use the correct conditional distribution in forming expectations is Muth's rational expectations hypothesis. In this way, we have a notion of equilibrium that not only requires that decisions are optimal and markets clear, but also that agents evaluate expectations in a way that is consistent with the model they are using.

Brock and Mirman (1972): Optimal Economic Growth and Uncertainty - The Discounted Case

The introduction of uncertainty about technological progress was one of the fundamental steps in the development of DSGE models. The paper by Brock and Mirman (1972) does so in the context of the Cass-Koopmans optimal growth model. Because of its discrete-time dimension, and because the random shock to technology enters in a very general form[4], the paper has become a classic in modern macroeconomics, so that most DSGE

[4]The formulation of the model is quite general in the sense that no specific assumptions are made on the form of the production function.

models are actually built on the foundations laid by Brock and Mirman.

In the Cass-Koopmans model the optimal consumption policy is the stable branch of the saddle point solution of the system of equations, which govern the dynamics of the model. However, the stable branch solution is a knife-edge policy, because any perturbation results in instability and eventual annihilation. This potential problem is what motivates Brock and Mirman to extend the optimal growth model to uncertainty. In fact, if the qualitative results of the theory are not changed after the introduction of uncertainty, then the deterministic growth model can be justified as the approximation of a more general model.

The type of uncertainty that Brock and Mirman consider is an error in observating output or an error in aggregation. This is done by introducing a random element in the production function $f(k, r)$, where k is the capital-labour ratio and r is the random variable. The variable r might also be thought of as an observation error on the capital stock. Because production is a random variable, consumption streams are random as well, and therefore the criterion of maximising the sum of discounted utilities is no longer valid and must be updated. Brock and Mirman choose the criterion function according to the theory of optimal decisions under uncertainty, thus assuming that individuals maximise the sum of discounted expected utilities. In this framework the maximization problem becomes:

$$\max \quad E\sum_{t=0}^{\infty} \delta^t\, u(c_t) \ ,$$

subject to:

$$c_t + k_t = f(k_{t-1}, r_{t-1}) \ .$$

The theory of dynamic programming can then be invoked to show that there exists a unique optimal consumption policy $\bar{c}_t$, which at any time t is a function of the state of the economy:

$$\bar{c}_t = g\left(f\left(\bar{k}_{t-1}, r_{t-1}\right)\right) \ .$$

The function g and the assumptions on r determine the properties of the stochastic process for the optimal capital-labour ratio $\bar{k}$. In particular, $\bar{k}$ follows a Markov process.

The distribution function of the optimal $\overline{k}$ is obtained from the transition function of the Markov process. Brock and Mirman's contribution is fundamental because they show how to extend the notion of steady state to the stochastic case. In the deterministic case the steady-state capital stock remains constant from period to period. In the stochastic case, the distribution of capital stocks remains constant from period to period.

Lucas and Prescott (1974): Equilibrium Search and Unemployment

"Expectations and the Neutrality of Money" was the first paper that situated the problem of expectation-formation in a general equilibrium setting. In this way, the expectations of future prices that agents form when, for example, deciding between labour and leisure are consistent with the expectations formed when buying or selling bonds, because they are the same. Aggregate variables are the result of decisions that are mutually consistent.

In "Expectations and the Neutrality of Money" money has real effects because of price-misperception, but the model cannot deliver a supply function that can be integrated with an IS-LM-type aggregate-demand theory. What it offered was an entirely new paradigm: one where the behaviour of one sector cannot be worked out without reference to that sector's interactions with the rest of the economy. It also constituted an example of a macroeconomic model where aggregative relationships come from the explicit modelling of microeconomic behaviour.

The general equilibrium approach, and the postulate that a macroeconomic model must be the aggregation of a microeconomic model can also be found in "Equilibrium Search and Unemployment" (1974), written with Edward C. Prescott.

The economy is constituted by a large number of spatially distinct markets or "islands". Product demand in each market shifts stochastically, driven by shocks that are independent across markets, but autocorrelated in each market. Labour in the only input used in production, and total labour force, allocated across markets, is constant. In each market, a worker may either work at the wage rate that is offered in that market, or leave. If she leaves, she earns nothing in the current period but enters a pool of unemployed workers that are distributed across the "islands".

At the beginning of the period, each market has a fixed workforce y. Define λ as the expected present value of job search, the same for all islands. New workers will arrive next period up until:

$$\beta \int_S v\left(s', y+a, \lambda\right) f\left(s', s\right) ds = \lambda \ ,$$

where β is the discount factor, v is the value of the expected future wage stream, $s, s' \in S$ are the current and expected future demand shocks in the market, with joint density f, and a are the new arrivals.

The equation above constitutes an equilibrium condition, from which is possible to derive the distribution of workers ψ for a given s, y and λ. Unemployment is then given by aggregating across markets:

$$u = \int_S \int a\left(s, y, \lambda\right) \psi\left(s, y, \lambda\right) ds\, dy \ .$$

Lucas (1976): Econometric Policy Evaluation: A Critique

General equilibrium models, with explicit micro-foundations are often used as a framework to conduct macroeconomic policy evaluation. Being optimising models, they carry with them the evaluation criterion (welfare maximisation) for policy that aggregative or Keynesian models lack. Moreover, they only have structural parameters which are invariant to policy changes[5].

The problem of parameter instability was famously made clear by Lucas in "Econometric Policy Evaluation: A Critique" (1976). At the time of writing it, quantitative macroeconomic policy discussions were conducted within a framework entirely specified by econometricians. For example, one could write that the behaviour of the economy was described by the following system of difference equations:

$$y_{t+1} = F\left(y_t,\, x_t,\, \theta,\, \varepsilon_t\right) \ ,$$

where y_t is the vector of endogenous variables, x_t is the vector of exogenous forcing variables, and ε_t is a vector of independent, identically distributed shocks. The task of the econometrician was then to estimate F and the parameters θ from past data, while economists suggested forms for the individual equations, in the absence of any theory that aggregated the decisions of the individuals.

[5]Lucas observes that the assumption that preference and technology parameters are invariant seems reasonable, but not beyond all doubt, and thus must itself be verified empirically.

With the knowledge of F and θ, policy evaluation consisted of simulating the behaviour of the economy under alternative specifications for policy x_t, and then evaluating the desirability of the outcomes according to some criterion. However, Lucas notes that F and θ must come from optimal individual behaviour under a given policy, and there is no guarantee that they will remain unchanged under some alternative policy x_t.

Does the Lucas critique close the possibility of conducting any meaningful policy evaluation exercise? Consider the policies that can be expressed as functions of the state of the system: a Taylor rule may belong to this category. Assume that they can be written as:

$$x_t = G(y_t, \lambda, \eta_t) \ ,$$

where G is known, λ is a fixed parameter vector, and η_t is a vector of disturbances. Then the remainder of the economy follows:

$$y_{t+1} = F(y_t, x_t, \theta(\lambda), \varepsilon_t) \ ,$$

that is, the parameters θ vary systematically with the structural parameters λ. The task of the econometrician is then to estimate the function F and the relationship between θ and λ.

However, in a DSGE model F and the relationship between θ and λ are obtained by aggregating the individuals' decisions according to the model. In this case λ are not just parameters describing policy but also preferences or technology. Estimates of λ can be obtained by various methods, but notice that parameters are structural (and the evaluation exercise meaningful) only if they are invariant to policy. This is hardly the case of, for example, the frequency at which prices are adjusted, since it is reasonable to expect that price-setters would want to increase the frequency of price adjustments in periods of high inflation. The DSGE methodology could accommodate this scenario by treating the frequency of price adjustment as an endogenous variable, which in equilibrium would depend, among the other things, on the parameters of the Taylor rule.

Lucas (1977): Understanding Business Cycles

In his introduction to "Studies in Business-Cycle Theory" (1981), Lucas wrote: "My own research has been concerned almost exclusively with the attempt to discover a useful theoretical explanation of business cycles". Moreover, again according to Lucas, to understand business cycles means to construct "a fully articulated artificial economy which behaves through time so as to imitate closely the time series behavior of actual economies" [6].

However, economics is characterized by the coexistence of several theories, originally formed to illuminate different aspects of economic behaviour. The simultaneous existence of several theories and models shows by itself that the selection criterion of "imitating closely" the data is not followed by all, as economics is also a normative science. As a result, the "attempt to discover a useful theoretical explanation of business cycles" is a rather vague research proposal as it does not indicate by itself a specific methodology or theory. Which model then can best explain business cycles, and which are the facts to be replicated? Moreover, given that our understanding of the economy is limited, do we want to select models that explain how the economy works or how it should work?

One could go back to the models that have been summarised above, and try to infer Lucas' proposed methodology for business cycle modelling from his most influential papers published between 1969 and 1977, summarised above. The dominant feature that they have in common is that prices and quantities are determined in equilibrium, thus agents do not have an incentive to change their behaviour. The papers summarised above illustrate Lucas' point of view that there is no serious alternative to the equilibrium method. To be precise, while "Rational Expectations and the Theory of Price Movements" is a contribution to Keynesian macroeconomics, the remaining ones illustrate Lucas' shift towards a general equilibrium point of view, and his final critique of Keynesian macroeconomics. In Lucas' view, the formulation of economic theories in explicit aggregate terms had not been beneficial, because it diverted the research effort away from the sources of institutional instability, favouring policies targeted at achieving a given level of output.

Lucas concludes that the empirical evidence points to the conclusion that business cycles are all alike, that is, the qualitative features of business cycles are the same for all countries. Therefore, Lucas argues, there exists the possibility of a unified explanation of business cycles, grounded in the general laws governing market economies, and not depending on the institutional features specific to particular countries. Moreover, if the features of business cycles are the same also across time, then the assumption that agents form their expectations rationally is not unreasonable, because there must exist stable

[6] Lucas (1977).

arrangements for processing the information and reacting optimally, without systematic biases.

Lucas argues that money might have a prominent role among the possible causes of business cycles. Real effects of money may arise because of price misperception, since monetary shocks generate stochastic price variability, and the agents cannot distinguish with certainty between permanent and transitory movements in prices, as well as general and relative price movements.

This approach to modelling business cycles is subsequently applied by Lucas in "An Equilibrium Model of the Business Cycle" (1979). However, in that paper agents' demands are not explicitly derived from utility maximization, that is, the link between preferences and demand functions is not made explicit.

Kydland and Prescott (1982): Time to Build and Aggregate Fluctuations

The model by Kydland and E.C. Prescott (1982), is, then, the first general equilibrium, micro-founded model of the business cycle. It could be seen as the first DSGE model, the point of arrival of a theory that gradually came into being from Lucas' drive towards an equilibrium theory of the business cycle, and was made possible by the concepts and modelling advances introduced by Muth (rational expectations) and Brock and Mirman (uncertainty).

Kydland and E.C. Prescott narrow the problem of explaining business cycles down to the one of accounting for the volatility and the co-movements of selected macroeconomic variables. Their approach integrates growth and business cycle theory, and focuses on a "real" or "nonmonetary" explanation of the business cycle. The crucial features of the model are the assumption that more than one time period is required for the construction of new productive capital, and the assumption that the utility function is non-time-separable, so that the intertemporal elasticity of substitution of leisure is higher than in the time-separable case. These assumptions increase the size and the persistence of the responses to the exogenous shocks.

The representative household solves the following problem:

$$\max \quad E\sum_{t=0}^{\infty}\beta^{t}u\left[c_{t},\alpha\left(L\right)l_{t}\right] \ ,$$

where β is the discount factor, c is consumption, l is leisure, L the lag operator, and

$\alpha(L) = \sum_{i=0}^{\infty} \alpha_i L^i$. Current utility depends not only on current leisure, but also on past leisure choices.

Total resources are constrained as follows:

$$c_t + i_t \leq f\,(\lambda_t, k_t, n_t, y_t) \ ,$$

where i is total investment, f is the production function, λ is a shock to technology, k is the capital stock, n is labour input and y are inventories. The capital stock evolves according to:

$$k_{t+1} = (1-\delta)\,k_t + s_{1t} \ ,$$

$$s_{j,t+1} = s_{j+1,t} \ ,$$

where $s_{j+1,t}$ is the amount allocated at time t to the investment project in the jth stage from the last, since it is assumed that $J > 1$ periods are required to build new capital. It must be stressed that the ad hoc assumption of time-to-build and non-time-separability of preferences are not indispensable. As shown by Long and Plosser (1983), it is possible to obtain persistence and co-movements in the responses with more general assumptions.

Kydland and Prescott build on the foundations laid by Muth, Lucas and Brock and Mirman, but most importantly, they put forward a general equilibrium model and make it fully operational, generating predictions that can be compared with the empirical evidence. Having obtained a numerical solution by means of approximation, Kydland and Prescott estimate as many parameters as possible from microeconomic evidence. For example, the observation that investment projects take approximately two years to complete is used to pin down the parameter J. Having then restricted as much as possible the number of free parameters, Kydland and Prescott test the model's ability to match the variance of a set of variables, the covariances with real output, and the autocovariance of output.

Kydland and Prescott's decision not to test the model versus a VAR (page 1360) is motivated by the existence of measurement problems, and by the abstract, stylized nature of the model. They do not provide the "true" model, but an approximation that can be used to answer a limited set of questions. They decide to focus on moments such as variances, covariances and autocovariances, for which the noise introduced by approximation and measurement errors is likely to be relatively small. The ability to match the actual moments with the ones generated by the model constitutes the test of the theory.

Another pioneering work in the field of DSGE models is the one from Long and Plosser (1983), published a year after Kydland and Prescott's work. Similarly to Kydland and Prescott, Long and Plosser show that the growth model can be modified to produce fluctuations that resemble the ones in the real economy, but they explore a different way. In order to replicate the moments in the data, Kydland and Prescott relied on the high intertemporal substitutability of leisure (which generates strong responses of labour and output) and on the assumption that capital requires "time to build" i.e. investment is not immediately productive (which introduces persistence in the model). Instead, Long and Plosser highlight the interactions among sectors. They demonstrate that the same properties can be obtained by assuming that consumers value a variety of goods, and they want to spread any unanticipated wealth increment over both time and commodities (including leisure). At the same time, Long and Plosser show that the large range of substitution opportunities limits the size of relative price changes, so that a shock in one sector moves current and future outputs in all sectors. In this way, the shock is propagated forward in time (generating persistence) and across commodities (generating co-movement in the time series).

Recents advances

In recent years, the DSGE methodology has proved to be a rigorous yet flexible tool, capable of both generating a better understanding of "old" problems, and of introducing new questions for research. As a result, the literature on DSGE models has witnessed an enormous expansion[7], so the following list of references is only indicative.

DSGE models have been built to analyse economic policy and time consistency issues (Kydland and Prescott 1977), asset prices (Lucas 1978), welfare and the cost of business cycles (Lucas 1987), and the analysis of business cycles has also been extended to open economies (Backus, Kehoe and Kydland, 1995), and the household sector (Benhabib,

[7] Apparently, the acronym DSGE was decided in a poll initiated by David Backus (1999). At the time, it was noted that the RBC acronym was too often used for a methodology that was "applied to models that have nothing real and may not even be about real business cycles". A brief history of the poll is available online at http://dge.repec.org/vote.html.

Rogerson, and Wright 1991, and Greenwood and Hercowitz 1991).

Moreover, with the introduction of heterogeneous agents (İmrohoroğlu 1989), DSGE models became capable of addressing questions that range from wealth distribution to the size of precautionary savings (Aiyagari 1993). There are also models that include non-Walrasian features such as efficiency wages or non-market clearing wages (Danthine and Donaldson 1995), and models with imperfect competition (Rotemberg and Woodford 1995). By considering quasi-geometric discounting, the DSGE literature has also incorporated some recent advances in behavioural studies (Krusell and Smith 2003).

While the earlier models focused on "real" explanations of business cycle fluctuations, much of the modern DSGE literature focuses on short-run fluctuations caused by monetary shocks. Money has been introduced in the models, both without nominal frictions (Cooley and Hansen, 1995), and with nominal frictions (Chari, Kehoe and McGrattan, 2000). DSGE models with monopolistic competition and nominal frictions have become the standard tool for the analysis of monetary policy, leading to a new approach, called the "New Keynesian science of monetary policy" (Clarida, Galí and Gertler 1999), which has replaced the IS/LM or AS/AD constructs.

All these recent advances extend the basic framework, either by including many sectors or several sources of shocks, or by introducing market imperfections or heterogeneity, or by enlarging the set of variables among which agents optimize. However, we can see that none of these recent advances would have been possible without the methodology that emerged from the seminal papers that have been individually discussed in this Section.

1.2 Main features of the DSGE methodology

The use of the DSGE methodology to obtain quantitative results and analyse a wide range of macroeconomic problems is now widespread, therefore the literature is continuously growing, and it is possible to gain still more insights by applying the methodology to a larger set questions. However, although the DSGE methodology has demonstrated to be very flexible, capable of incorporating a variety of assumptions, it remains unique, as its key features have remained constant.

The equilibrium method is one of these features. In fact, some of the recent DSGE models accommodate elements of the Keynesian approach (such as price rigidity), therefore they do not rely on the intertemporal substitution of leisure (as in Lucas and Rapping

1969) or on price-misperception (as in Lucas 1972) to generate fluctuations, but they share with these two papers the assumption that all markets clear. Another key feature of the DSGE methodology is that it specifies the individuals' dynamic optimisation problem(s) under uncertainty, moreover, being general equilibrium, it considers all sectors and all transactions between different kinds of individuals.

The literature has relied heavily on both the assumption that agents form their expectations rationally and on Muth's notion of rationality, but it seems prudent not to proclaim them as one of the key features of the DSGE methodology. On the one hand, it is difficult to say what "rationality" means, whether the meaning attributed to it by Muth is unique, and if not, whether other notions of rationality may be used in DSGE models. On the other hand, the very recent developments in behavioural sciences, which find that individuals use some personal heuristics rather that "rational optimization" in their decision-making, will probably influence macroeconomic research. However, drawing from the spirit of the Lucas' critique, it is hard to see a role in macroeconomics for models in which agents are systematically biased, at least until the relationship between decision-making and external influences or policies is fully understood.

In any case, as these future developments have yet to materialise, the DSGE methodology still remains a unique way to formulate and solve macroeconomic problems, a way that is very different from the aggregative models that dominated macroeconomics until two decades ago. A complete evaluation of DSGE modelling, based on its adaptability to solve all possible problems, would require more than a thesis. However, the experience gained by applying several DSGE models to the study of fluctuations can be useful to attempt an evaluation of this unique methodology. Although this exercise is limited to the analysis of economic fluctuations, and it considers only the particular viewpoint of the questions that are analysed in the thesis, it can give valuable insights because of its applied nature.

Economic phenomena are very complex because, generally speaking, they are caused by numerous factors, moreover, the factors that determine the observed events are not always directly observable and measurable. As a result, the criterion of comparing a model's predictions with the data typically leaves economists with not just one, but several theories to choose from. In this situation, the DSGE methodology has provided a guide to organize thoughts (and it has itself generated new questions for research). Certainly this "value added" can be appreciated by applying the methodology directly to some problems.

As mentioned in the introduction, the forthcoming chapters consider several issues related to the analysis of economic fluctuations, and in doing so they consider several modelling approaches. In fact, while the models in Chapters 2 and 3 are within the RBC theory, the models of Chapters 4 and 5 are DSGE models with monopolistic competition and sticky prices, close to the recent "New Keynesian" approach, which was cited before. This change in the modelling approach, or in the theory used, is helpful to get an overall perspective for a "tentative" or provisional evaluation of the DSGE methodology, which will be finally given in the Conclusion chapter of the thesis.

Chapter 2

Business Cycles in Italy: Facts and Models

2.1 Introduction

Since the early 1980s, one of the main challenges of modern macroeconomic research has been understanding and reproducing in a model economy the cyclical fluctuations of the main macroeconomic variables. The real business cycle theory (RBC) hinges on both neoclassical theory and dynamic general equilibrium theory to accomplish this task [1]. However, the cyclical behaviour of the main macroeconomic variables can be different across countries. This chapter documents first the business cycle facts for the Italian economy, and then tries to quantify how much they can be explained by standard RBC theory.

Most of the time, the RBC literature has focused on the US, and the cyclical properties of the US economy have provided the yardstick for the evaluation of RBC theory. However, the relevance of RBC theory could be strengthened by showing that it is also able to explain other countries' cyclical fluctuations, even when fluctuations are very different from the US. Italy is an interesting "test" case because fluctuations in labour input are strikingly different from the US. In Italy, hours fluctuate much more than employment, while in the US the opposite is true.

[1] Since the early work of Kydland and Prescott (1982) the framework has been considerably expanded to account for monetary shocks as well as real shocks, in models with or without Keynesian features such as wage or price rigidities. As a result, to account for both real and monetary models, many people now prefer to refer to this approach as dynamic stochastic general equilibrium modelling (DSGE).

Little research has been conducted on explaining business cycles in Italy. Apparently, there are only two papers that calibrate a RBC model for Italy: Censolo and Onofri (1993), and Maffezzoli (2001).

Censolo and Onofri's paper is an application to Italy of the model of King, Plosser and Rebelo (1988). They find that the model cannot replicate the high volatility of hours and the negative or null correlation of hours and wages in the data[2]. The paper estimates hours of work by extending to all sectors the hours worked in the industry sector, subject to an adjustment for employment and the contractual workweek in each sector[3]. Censolo and Onofri do not consider, however, hours and employment data separately, and their measure of the labour input is likely to be a combination of hours and employment ratios.

Maffezzoli develops a DSGE model with endogenous growth and equilibrium unemployment, generated respectively by learning-by-doing and monopolistic unions, and he calibrates it to reproduce several long-run features of the Italian economy. Compared to the indivisible labour model[4], Maffezzoli's model with monopolistic unions is better suited to explain the Italian business cycle. Although both models have only a limited ability to reproduce the cyclical fluctuations in the Italian economy, the correlation coefficients between the observed and simulated series are higher for the monopoly union model. Moreover, because it includes a human, not only physical, capital stock, Maffezzoli's monopoly union model generates responses to productivity shocks that are more persistent.

However, Maffezzoli does not explicitly consider labour adjustment along the intensive margin, that is, in his model the labour input is measured only by the employment rate. As a result, in his model it is not possible to ascertain whether the RBC theory can explain the high volatility of hours in the Italian data. Moreover, the omission of hours from the production function is likely to introduce a bias in the estimation of the Solow residuals.

This chapter collects detailed information on the cyclical fluctuations of the Italian economy, and then tries to quantify how much these can be explained by standard RBC models. First, Italy is modelled as a closed economy, and then, by introducing the possibility of borrowing and lending from the rest of the world at an exogenous interest rate, as a small open economy. In both models, utility is separable in consumption and leisure,

[2]More on this problem can be found in Hansen and Wright (1992).

[3]Censolo and Onofri further adjust their measure of hours worked by regressing it on a constant plus agricoltural employment, in order to filter out seasonality.

[4]Hansen (1985).

with different parameters for the intertemporal elasticities of substitution. As a result, Hansen's (1985) indivisible labour model becomes a special case.

To begin with, the focus of the investigation is primarily to assess the ability of the model to replicate the high volatility of hours in Italy. In both the open and the closed economy models of this chapter, households adjust their labour supply along the intensive, not the extensive, margin. In the next chapter, households adjust labour supply along both margins.

In both the closed and open economy versions, the standard RBC models are not able to generate a standard deviation of hours worked approximately twice as large as the standard deviation of output. As the information on hours worked in Italy is unfortunately limited to large industrial firms, this benchmark value must be taken with some care, but it cannot be ignored altogether. Thus the trouble for the standard RBC model is that, for sensible parametrizations, within the range of values found in the literature, it cannot generate a standard deviation of hours relative to output higher than 70%, as this chapter will explain. As a result, the standard RBC model is of limited applicability to the Italian case. However, the standard model was designed for the US, and it is therefore possible that a RBC model modified to take into account the institutional features of the Italian economy may generate the required volatility of hours.

This chapter is organised as follows. Section 2.2 reports the main facts of the Italian business cycles, presenting a set of statistics extensively used in the RBC literature, with the aim to have a definite benchmark for comparison. Section 2.3 presents and evaluates a standard closed-economy RBC model, and Section 2.4 models Italy as a small open economy. Section 2.5 concludes.

2.2 The Main Facts of Business Cycles in Italy and the US

The RBC literature has focused on the aggregate fluctuations of the US economy, so it comes as no surprise that there are extensive accounts of the cyclical properties of the US[5]. Such accounts can also be found for other countries, but statistics for the Italian economy are not as abundant as for the US. In particular, while the main macroeconomic time series are available for the Italian economy, there is limited information on hours worked

[5] A very detailed account is provided, among others, by Stock and Watson (1999).

in Italy. However, information on hours is crucial to calibrate and test RBC models[6].

After the work of Kydland and Prescott (1990), a standard procedure for defining the empirical regularities of business cycle fluctuations that the RBC theory tries to replicate has taken hold in the literature. Kydland and Prescott emphasized that the choice of facts to report should be guided by neoclassical theory, and they advocated the provision of the following information for each variable: amplitude of fluctuations (standard deviation), degree of co-movement with GDP (contemporaneous cross correlation), and phase shift (cross correlation at different lags and leads). Another important feature is the degree of persistence (first-order autocorrelation) in a series. In short, RBC theory is interested in second-order moments only, leaving aside the issue of identifying the underlying trend along which the series fluctuate[7].

This chapter tries to fill the informational gap for the Italian economy[8]. It is also necessary to make an explicit comparison with the US, since models that proved successful in dealing with American fluctuations might not produce desirable results when the economy under scrutiny is a different one.

Except for employment and hours worked, all the statistics for Italy are calculated using OECD data for the period 1970(I) to 1998(III)[9]. Hours worked in Italy come from Istat[10] and refer to blue-collar workers in industrial firms with more than 500 employees[11]. Given that those are the only data on hours worked in Italy, they are taken as indicative of hours worked by all employed people. Wages and productivity calculations are therefore not entirely based on OECD data. In addition to this, employment data for Italy does not come from the OECD but from the Bank of Italy[12]. The US statistics are calculated using OECD data for the same period, 1970(I) to 1998(III), in order to ensure comparability

[6]This is because the transmission of shocks in a RBC model depends decisively on the intertemporal substitution of labour. The RBC transmission mechanism is explained, for example, in Lucas (1987) pp. 41-42. The idea that workers substitute labour intertemporally was originally due to Lucas and Rapping (1969), before appearing in RBC theory.

[7]The sort of decomposition used is not an irrelevant issue, as it can naturally affect the results obtained. A common choice is the Hodrick-Prescott (HP) filter. The HP filter is a two-sided linear filter that minimizes the variance of the series around the calculated trend. One of the advantages of the HP filter is comparability with other RBC studies.

[8]Accounts of business cycle regularities for other countries can be found in Blackburn and Ravn (1992), and Christodoulakis, Dimelis and Kollintzas (1994), and especially in Ravn (1997) for business cycles at the international level.

[9]Earlier data for Italy are not available from the OECD.

[10]They are published first in the Indicatori Mensili series, then in the Bollettino Mensile di Statistica. Data available and fit for use are from 1973(I) to 1995(IV) only.

[11]According to Istat, the firms in the sample employ the 21.9% of the workforce in their sector, which corresponds to the 85.9% of the workforce in large industrial firms.

[12]Employment data from the OECD displayed a disproportionately high volatility.

between the two countries. Since data on hours worked is not published by the OECD, hours worked in the US come from the Bureau of Labor Statistics and are based on the Establishment Survey. A description of the definitions used and the source of the data is in Table 2.12.

All statistics refer to seasonally adjusted[13] quarterly data (in logs), expressed in constant 1990 prices whenever necessary, and detrended using the Hodrick-Prescott (1997) filter, with a smoothing parameter equal to 1,600. Except for the current account and inventory investments, which are expressed as ratios to GDP, data used are in logarithms. For the sake of clarity of the exposition, data on aggregate variables are grouped into two broad categories: those referring to production and those referring to expenditure or demand components.

Production inputs

Tables 2.1 and 2.2 report the main statistics that describe the cyclical properties of production inputs: standard deviations in percentage and relative to output, first-order autocorrelations, and cross-correlations with output at different lags and leads. A positive sign indicates that the series is procyclical, a negative sign indicates that the series is countercyclical, a number close to zero indicates that the series has no correlation with the cycle. If the maximum correlation is reached at time t + i (t - i), the cyclical component of the series tends to lag (lead) the cycle by i quarters.

The series for capital is constructed using a variant of the perpetual inventory method described in Barro and Sala-i-Martin (1995), page 348 [14]. Data on labour productivity and wages at quarterly frequencies are not available from the OECD, therefore they have been derived using, respectively, the series for GDP and compensation of employees, both in the national accounts. Compensation of employees is adjusted to take into account the earnings of self-employed workers, considered as entrepreneurial income[15] by the OECD. Only an approximate adjustment method can be adopted, and the explanation is in Table 2.12.

By looking at cross-correlations at different leads and lags, it can be inferred that cycles

[13] Difference from moving average, additive.

[14] The main difference is that it does not assume a constant depreciation rate. Instead, depreciation is measured using the series Consumption of Fixed Capital, from the OECD National Accounts.

[15] Since Italy has a large proportion of self-employed workforce, if this adjustment was not made then the resulting profit share in total GDP would become much higher than in the US.

Table 2.1: Italy - Production inputs

	% st dev	Rel st dev	1-st AC	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)
				Cross-correlation of output at time t with						
GDP	1.44	1.00	0.84	0.23	0.56	0.84	1.00	0.84	0.56	0.23
Capital Stock	0.38	0.26	0.91	-0.57	-0.44	-0.19	0.12	0.39	0.56	0.61
Labour input:										
Employment	0.81	0.56	0.74	-0.29	-0.08	0.13	0.32	0.46	0.50	0.49
Per capita hours	2.94	2.04	0.54	0.22	0.38	0.56	0.67	0.41	0.13	-0.07
Earnings:										
Wages per capita	1.82	1.26	0.33	0.12	0.24	0.32	0.33	0.22	0.09	-0.01
Wages per hour	2.95	2.05	0.50	-0.15	-0.21	-0.33	-0.44	-0.27	-0.08	0.08
Labour productivity:										
Output per capita	1.40	0.97	0.78	0.40	0.62	0.78	0.84	0.60	0.28	-0.05
Output per hour	2.51	1.74	0.39	-0.04	-0.12	-0.25	-0.35	-0.14	0.04	0.11

in the US have longer duration than in Italy. The volatility of output is bigger in the US than in Italy, and the first-order autocorrelation is not very different. As it is a stock variable, capital is characterised by the lowest volatility and the highest autocorrelation. The cross correlations of output and capital in the two countries have similar patterns.

Employment is much more volatile, less persistent and more procyclical in the US than in Italy. Employment lags the cycle in both countries. The behaviour of hours also differs a lot between the two countries, however it has been already pointed out that employment and hours data do not come from a common source[16]. Hours are a lot more volatile, less persistent and less procyclical in Italy than in the US. Not surprisingly, since data on earnings and average labour productivity are derived from data on employment and hours, statistics on earnings and average labour productivity also differ a lot between the two countries. Wages per hour and output per hour are countercyclical in Italy, but wages and output per employed are not.

In order to understand whether the volatility of hours worked in Italy is high also in comparison with other countries, Table 2.3 reports some international statistics. Except

[16]The high volatility of hours in Italy may be due to the sample, because the only data available is for industrial firms with more than 500 employees. Overall, hours worked in Italy may be more or less volatile than the data shown in Table 2.2. For example in the US, while the standard deviation of hours worked by production workers is 0.39%, but the standard deviation of hours worked in industry is 0.91%.

Table 2.2: USA - Production inputs

	% st dev	Rel st dev	1-st AC	Cross-correlation of output at time t with x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)
GDP	1.70	1.00	0.87	0.48	0.69	0.87	1.00	0.87	0.69	0.48
Capital Stock	0.43	0.25	0.94	-0.28	-0.10	0.11	0.36	0.56	0.71	0.80
Labour input:										
Employment	1.31	0.77	0.65	0.24	0.40	0.56	0.71	0.76	0.69	0.56
Per capita hours	0.39	0.23	0.74	0.39	0.57	0.70	0.75	0.54	0.26	0.05
Earnings:										
Wages per capita	1.25	0.74	0.87	0.65	0.65	0.58	0.45	0.25	0.06	-0.13
Wages per hour	1.10	0.65	0.82	0.60	0.53	0.41	0.24	0.10	-0.03	-0.17
Labour productivity:										
Output per capita	1.04	0.61	0.34	0.33	0.41	0.45	0.45	0.25	0.15	0.05
Output per hour	1.20	0.71	0.47	0.42	0.54	0.62	0.64	0.39	0.22	0.06

for Canada, all the data is from a common source, Eurostat, and it is collected under the European Union Labour Force Survey (EU LFS). Because the concepts and guidelines used under the EU LFS are the same for all countries, there are little comparability problems, but unfortunately the Eurostat data has two main limitations. Firstly, Eurostat quarterly series are available only from the mid or late nineties[17]. Secondly, the survey is conducted at the household, not at the firm, level, and it has been noted that households who are interviewed do not often make a distinction between actual and usual hours of work. Thirdly, the series for Italy are characterised by two abrupt falls in hours worked, which may be due to breaks in the series. For these reasons, it is better to use the information on Italy given by Table 2.3 not as a substitute, but as a way to understand better the data reported on Table 2.1.

However, one advantage of the Eurostat statistics is that the series on hours are disaggregated by economic activity[18] and by type of occupation. This level of detail may conduct to a better understanding of the high volatility of hours worked in Italy reported by Table 2.1. The data on hours of Table 2.1 is obtained from Istat, and it relates to blue collar workers in large industrial firms, which may or may not be representative of the

[17]Therefore, it was possible to find long series only for Canada, from 1987 onwards.

[18]NACE, Classification of Economic Activities in the European Community.

Table 2.3: International comparisons - Per capita hours

	% standard deviation		
	Employment, total	Employment, industry	Employees, Plant & machine operators and assemblers
Belgium	0.57	0.70	1.04
Canada	1.79 (1)	.	.
Denmark	0.96	1.12	1.57
Finland	0.77	0.98	1.04
Greece	0.94	0.95	0.75
Italy	1.09 (2)	1.14 (2)	0.96 (2)
Spain	1.01	1.22	1.15
United Kingdom	0.39	0.44	0.55

Source: all Eurostat, except Canada (Statistics Canada). All data is quarterly, seasonally adjusted and detrended using the HP filter.

(1) : calculated by dividing the total number of actual hours worked by the total number of employed persons.

(2) : excludes the second quarter of 2002 and the first quarter of 2003.

whole economy.

The aim of Table 2.3 is to compare the volatility of hours in other countries, both in the aggregate and at the same level of sectoral and occupational disaggregation as in the Italian data. If this comparison is possible, then we could infer by analogy whether the high volatility of hours is characteristic of the whole Italian economy or of the sectoral and occupational disaggregation, although this sort of exercise can increase understanding only in a very intuitive manner, since an analogy cannot be a substitute for the missing facts.

The most interesting result that emerges from Table 2.3 is that the UK is the only country where the volatility of hours is similar to the US, while in all the other countries the volatility is a lot higher, although not as much as in Italy.

Unfortunately, for the other countries it was not possible to find data on blue collar workers in large industrial firms, since the Eurostat data is disaggregated by either economic activity or type of occupation, but not by both combined. However, it seems reasonable to assume that many of the occupations which can be classified under "Employees, Plant and machine operators and assemblers" are peculiar only to the industry sector, so the last column of Table 2.3 is probably the closest, for the level of detail, to the available Italian data.

In 4 countries[19] out of 7, per capita hours of plant and machine workers fluctuate more than industry hours, which themselves are more volatile than in the aggregate economy. However, within the same country the increase in the standard deviation at the disaggregated level is sizeable, but not huge. So, the intuition that can be drawn from Table 2.3 is that the 2.94% standard deviation of hours in Italy is likely to overestimate the true aggregate volatility, but even if we had data on the whole economy we would probably find a standard deviation of hours a lot higher than in the US.

Expenditure components

Private consumption is more volatile in the US, and a little less persistent than in Italy. Consumption leads the cycle in the US, while in Italy the highest autocorrelation is the contemporaneous one.

The behaviour of government consumption is remarkably different in the two countries. Government consumption is far more volatile, and less persistent, in the US than in Italy, moreover, government consumption leads the cycle in Italy and exhibit negative correlations at t + i, while in the US there is almost no significant relation with the cycle except at the higher leads. These differences may be due to the political-economic cycle, which could differ between the two countries.

Total investment is the expenditure component with the highest degree of volatility in the two countries. Much of the volatility comes from inventories, since total investment (that is given by fixed investment plus inventories, see data definitions in Table 2.12) is a lot more volatile than fixed investment[20]. Both total and fixed investment are strongly procyclical in the two countries, but in Italy the tendency of investment to affect positively future deviations from trend output lasts only for two-three periods.

Current account to GDP is a lot more volatile in Italy than in the US, but about as persistent. It is countercyclical in both countries and also at different lags and leads, but in Italy it tends to affect positively future deviations of output from trend. By contrast, exports and imports are procyclical. Both have higher volatility and persistence in the US than in Italy. The movement of exports and imports with the cycle differs in the two countries. In the US exports lag positively the cycle, and the correlations of imports with

[19] Belgium, Denmark, Finland and the UK.

[20] If this seems less apparent in the statistics of inventory investment in Tables 3 and 4, it is because those statistics are not for logarithms but for ratios on GDP, since the logarithmic transformation is not feasible for data with negative values.

Table 2.4: Italy - Expenditure components

	% st dev	Rel st dev	1-st AC	Cross-correlation of output at time t with x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)
GDP	1.44	1.00	0.84	0.23	0.56	0.84	1.00	0.84	0.56	0.23
Consumption:										
Private consumption	1.25	0.87	0.90	0.28	0.52	0.71	0.80	0.77	0.62	0.41
Government cons.	0.54	0.37	0.75	0.23	0.10	0.01	-0.06	-0.09	-0.12	-0.12
Total consumption	0.97	0.68	0.91	0.31	0.52	0.69	0.78	0.74	0.59	0.39
Investment:										
Total investment	6.92	4.81	0.76	0.10	0.43	0.73	0.89	0.76	0.52	0.20
Fixed investment	3.66	2.55	0.90	0.17	0.40	0.63	0.78	0.78	0.68	0.50
Inventory inv. / GDP	1.18	0.82	0.66	0.00	0.32	0.61	0.73	0.56	0.28	-0.06
Trade:										
Current account / GDP	1.20	0.83	0.77	0.05	-0.21	-0.45	-0.58	-0.60	-0.54	-0.37
Exports	3.94	2.74	0.53	0.23	0.28	0.33	0.32	0.17	-0.07	-0.24
Imports	4.80	3.34	0.67	0.31	0.49	0.67	0.73	0.63	0.40	0.13

output stay high at different lags and leads. Also, in the US exports lead negatively the cycle, while in Italy they tend to affect positively future output, and this may be due to higher degree of openness of the Italian economy[21]. The different cycle length may explain why in Italy the correlations of imports with output are quite short-lived, if compared with those in the US.

2.3 A Closed Economy Model

In this section a standard RBC model is presented and calibrated for Italy, in order to test its ability to capture the business cycle statistics shown in the previous section.

The economy is populated by a large number of identical agents, households and firms. Utility is separable in consumption and leisure, and it is possible to check if the model can reproduce Italian business cycles under different values for the intertemporal elasticity of substitution of leisure. The latter parameter can generate very different values for the standard deviations of hours and output.

[21]The log transformation implies that all cyclical deviations from trend are in percentage terms, and not in absolute terms.

Table 2.5: USA - Expenditure components

	% st dev	Rel st dev	1-st AC	Cross-correlation of output at time t with x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)
GDP	1.70	1.00	0.87	0.48	0.69	0.87	1.00	0.87	0.69	0.48
Consumption:										
Private consumption	1.37	0.81	0.88	0.68	0.81	0.89	0.87	0.69	0.51	0.30
Government cons.	0.82	0.48	0.58	-0.09	-0.12	-0.14	-0.07	0.00	0.09	0.19
Total consumption	1.07	0.63	0.87	0.66	0.79	0.86	0.86	0.69	0.52	0.33
Investment:										
Total investment	6.65	3.92	0.83	0.45	0.63	0.79	0.93	0.81	0.60	0.37
Fixed investment	4.73	2.79	0.90	0.48	0.68	0.85	0.95	0.87	0.71	0.52
Inventory inv. / GDP	0.56	0.33	0.48	0.26	0.34	0.46	0.61	0.46	0.24	0.01
Trade:										
Current account / GDP	0.45	0.27	0.78	-0.48	-0.46	-0.46	-0.44	-0.38	-0.22	-0.06
Exports	4.39	2.59	0.78	-0.30	-0.12	0.10	0.33	0.45	0.52	0.53
Imports	5.34	3.15	0.81	0.54	0.63	0.74	0.80	0.72	0.52	0.30

The model

All variables are in per capita terms, and there is no population growth. Each household seeks to maximise her expected utility over infinite sequences of consumption $\{c_t\}_{t=1}^{\infty}$ and leisure $\{l_t\}_{t=1}^{\infty}$ pairs:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] .$$

Households allocate their time between productive activities and leisure, and the total amount of time available to them is normalised to 1: $h_t + l_t = 1$. They own the capital stock and rent it to the firms. The household budget constraint is:

$$c_t + i_t = w_t h_t + (\delta + r_t^k) k_t .$$

Firms produce output y_t according to a Cobb-Douglas production function, which exhibits constant returns to scale in capital k_t and hours h_t:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha} .$$

Uncertainty comes from the exogenous stochastic process z_t. It is assumed that z_t

follows an AR(1) process:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \ ,$$

with ε_t i.i.d $N\left(0, \sigma^2\right)$.

The law of motion for capital is:

$$k_{t+1} = (1 - \delta)\, k_t + i_t \ ,$$

where δ is the depreciation rate. Capital at time zero, k_0, is given. The resource constraint for the economy as a whole is:

$$c_t + i_t = z_t k_t^{\alpha} h_t^{1-\alpha} \ .$$

As there are neither taxes nor other distortions, the allocations in a decentralised economy with perfect competition are the same as those resulting from the solution to the optimisation problem faced by a social planner.

Factor demand comes from the firms, which maximise profits. The rental rates of labour and capital are given by, respectively:

$$w_t = (1 - \alpha)\ z_t k_t^{\alpha} h_t^{-\alpha} \ , \tag{2.1}$$

$$r_t^k = \alpha\, z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta \ . \tag{2.2}$$

Households own all factors of production and choose optimally their supply of labour and capital. Optimality implies that these two efficiency conditions must hold:

$$\frac{w_t}{c_t^{\theta}} = A\,(1 - h_t)^{-\gamma} \ , \tag{2.3}$$

$$\beta\, E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left(1 + r_{t+1}^k \right) \right] = 1 \ .$$

These two conditions, together with the laws of motion of capital and the shock, the wage and interest rate equations, and the resource constraint of the household, determine the equilibrium path followed by the model economy. The model cannot be solved analytically. The behaviour of the model economy in response to the exogenous shocks is obtained using an approximated solution, based on the log-linearisation of all the equations around the deterministic steady state of the model.

Calibration

The choice of parameter values is made so that a set of values from the steady state equations match the corresponding time-series averages for the Italian economy. The same data set used for the statistics of Section 2.2 is used for calibrating the parameters, using real per capita variables. Nevertheless, some preference parameters cannot be estimated in this way because they do not enter the steady state equations, and there are no microeconometric studies for Italy that estimate them. Therefore, parameters are chosen according to the steady state equations whenever possible, otherwise a sensitivity analysis is preferred, in order to check how the model results change across different parametrizations.

The preference parameters that are considered for the sensitivity analysis are the coefficient of relative risk aversion θ, and the inverse of the intertemporal elasticity of substitution of labour γ. Both the values 1 and 2 are common choices for θ, so it is better to check the performance of the model, in terms of second-order moments, for both values. For the parameter γ, three cases are considered: γ equal to 1, 0 (infinite intertemporal elasticity of substitution, as in the indivisible labour model), and the intermediate case 0.5.

The remaining parameters are calibrated as follows. The depreciation rate δ is calculated by taking the average ratio of capital consumption over net stock of capital, which gives a quarterly value of 0.0088 for Italy[22]. The labour share of output $1-\alpha$ is calculated as the ratio adjusted compensation of employees over GDP, which gives a value for $1-\alpha$ equal to 0.67.

The discount rate β is calibrated from the steady state equation $\beta\left(1+r^k\right)=1$. The interest rate r^k can be calculated (once values for α and δ are given) from the output/capital ratio in the data, but this procedure is affected by measurement errors in the capital stock. However, without capital adjustment costs, and abstracting from risk, the return on physical capital (net of depreciation) must be equal to the return on financial assets. Therefore, the steady state interest rate is estimated using an average real inter-

[22]Data definitions and sources as in Table 7. According to the OECD, Consumption of fixed capital is the decline in value due to physical deterioration and obsolescence.

est rate from the financial markets. To ensure consistency with the next chapter (small open economy model, with an exogenous return on financial assets), I choose the German 3-month fibor. This financial rate, adjusted for the growth in consumer price index and averaged over time, gives a value for the steady state r^k equal to 0.0078. This gives a value for β equal to 0.99, in line with the literature.

The parameter A is given by the steady state equation:

$$A = \frac{(1-h)^{\gamma}}{c^{\theta}} \frac{y}{h} (1-\alpha) \ , \tag{2.4}$$

which can be rewritten as

$$A = (1-h)^{\gamma} \left(1 - \delta \frac{k}{y}\right)^{-\theta} (1-\alpha)\, h^{-\theta} \left(\frac{y}{h}\right)^{1-\theta} \ ,$$

where y and k are output and capital per person. The capital-output ratio and the ratio of output on hours are already given by the steady state equations:

$$\beta \left(1 + \alpha \frac{y}{k} - \delta\right) = 1 \ ,$$

$$\frac{h}{k} = \left(\frac{y}{k}\right)^{\frac{1}{1-\alpha}} \ .$$

These equations allow to calibrate A, once γ, α and h are given. The quantity h is the steady-state fraction of time that the individual devotes to market activities, which is calibrated at 0.12, given data on hours worked[23]. Table 2.6 summarizes the calibrated parameter and steady state values.

A consistent estimate of the exogenous process for technology could be obtained from the production function as follows:

$$\log z_t = \log y_t - \alpha \log k_t - (1-\alpha) \log h_t \ , \tag{2.5}$$

provided the information on output and factor inputs is free from measurement errors. However, as noted in Section 2.2, the only information on quarterly hours worked in Italy is

[23]The average hours worked in a week (35.53), divided by total time available (16 hours times 7 days a week), and multiplied by the fraction of the Italian population that works (0.37). In this way, the leisure enjoyed by the people who do not work is included in the leisure enjoyed by the representative agent of the model economy.

from industrial firms with more than 500 employees, which is an imperfect measure of hours worked in all sectors of the economy. A consistent estimate of the exogenous process z and the shock ε is important because the technology shock determines the stochastic properties of the series generated by the model. Therefore, in the absence of more information on hours worked in Italy, it is better to resort to Kydland and Prescott's (1982) strategy of choosing the standard deviation of the shock ε so that the model reproduces the standard deviation of output in the data[24]. Then, following this strategy, the test of the theory is whether there is a set of parameters for which the model's implied standard deviations measured in relative terms, with respect to the standard deviation of output, match the data. This approach is followed also by Baxter and Crucini (1993), and Correia, Neves and Rebelo (1995), among others.

Maffezzoli (2001) calibrates the Solow residuals z for the Italian economy by using standard units of labour from Istat, as an approximate measure of the labour input. According to ISTAT[25], a standard unit of labour represents the amount of labour that is supplied by a person that works full time. The labour supplied by an individual may therefore be more (for example, because the individual works only part-time), or less than a standard unit of labour (for example, because the individual has more than one job). Therefore, standard units of labour measure fulltime equivalent employment. In particular, measured standard units of labour in Italy include both regular and irregular (underground economy) workers, but they do not consider overtime hours and leaves of absence. Therefore, standard units of labour cannot constitute a measure of per capita hours of the same quality as survey data. What is lacking for Italy is information on hours for all workers, obtained at the microeconomic (individual or firm) level.

The transmission of shocks in a RBC model depends on the agents' choice of substituting labour intertemporally. This implies that information on hours is crucial to calibrate and test RBC models. The absence of information at the micro level on hours worked in Italy limits the possibility of understanding the Italian business cycle, and Kydland and Prescott's (1982) strategy of matching relative standard deviations seems to be the only viable choice. However, our approach will not discard the only information available for Italy, namely hours worked in industrial firms with more than 500 employees. As noted in

[24]The autoregressive coefficient of the AR process for z is chosen so as to match as close as possible the autocorrrelation of output. This implies a value for ρ of 0.99.

[25]Sistema europeo dei conti, SEC 95.

Table 2.6: Calibrated parameter and steady state values

δ	α	ρ	β	h
0.0088	0.33	0.99	0.99	0.12

Section 2.2, according to that series the standard deviation of hours in Italy is 7-8 times more than in the US. We believe that high volatility may characterise the behavior of hours of work in Italy. In fact, it is entirely probable that the rigidities in the Italian labour market prevent the adjustment of the labour input along the extensive margin, thus leaving the burden of adjustment to hours only. This view of the Italian labour market is put forward by other authors. For example, Marchetti and Nucci (2001) justify their empirical findings on the behavior of work effort over the Italian business cycle by a theoretical model where firms face hiring costs.

Since the high standard deviation of hours in Italy may be due to the rigidities in its labour market, the statistics shown in Table 2.1 are taken as a fact that we want to replicate. Moreover, the ability of the RBC model to generate simulated hours which have high volatility can be put to test. If the models succeed in this challenge, then their applicability to countries other than the US, where hours fluctuate much less than employment, can greatly increase.

Evaluation of the model for different parameter values

All the statistics for the closed economy were computed on logarithms of HP-detrended data, generated by simulating the model for 225 periods, throwing away the first 25 observations, repeating 100 times, and then computing moments as averages over repetitions[26].

The calibration of the model based only on the steady state equations leaves some crucial parameters free to be set at different values. Those parameters are the intertemporal elasticities of substitution of consumption and hours, which greatly affect the model performance in terms of the second-order moments and the response of the variables to a technology shock. Table 2.7 describes the effects of different choices of the parameters on the most important moments. In this way a sensitivity analysis is carried out, but instead of finely varying the parameters around a chosen value, a set of values is chosen, which

[26] Relative-volatility statistics may change in different simulations. Because of the certainty-equivalence property of the log-linear approximations, they do not depend on the variance of the innovations of the technology shock.

Table 2.7: Cross country correlations

	Relative volatility with respect to output			First-order autocorrelation			Contemporaneous correlation with output		
	c_t	h_t	w_t	c_t	h_t	w_t	c_t	h_t	w_t
$\theta = 1$									
$\gamma = 0$	0.33	0.68	0.33	0.72	0.69	0.72	0.97	0.99	0.98
$\gamma = 0.5$	0.33	0.62	0.38	0.73	0.70	0.72	0.97	0.99	0.98
$\gamma = 1$	0.35	0.58	0.42	0.73	0.70	0.72	0.98	0.99	0.99
$\theta = 2$									
$\gamma = 0$	0.33	0.37	0.65	0.72	0.71	0.72	0.99	0.96	0.99
$\gamma = 0.5$	0.34	0.31	0.71	0.70	0.70	0.70	0.99	0.96	0.99
$\gamma = 1$	0.35	0.27	0.75	0.72	0.72	0.71	0.99	0.95	0.99

reflects some parameter choices made in other RBC studies.

The case $\gamma = 0$ corresponds to the indivisible labour model of Hansen (1985) and it implies an infinite intertemporal elasticity of substitution of the labour input. When $\gamma = 1$ the intertemporal elasticity of substitution of labour is equal to 1, and $\gamma = 0.5$ is the intermediate case. When γ increases, the volatility of hours relatively to output decreases because households are less willing to substitute hours over time in response to the exogenous shock, and they become relatively more willing to substitute consumption instead of hours, so the volatility of consumption increases slightly[27]. The explanation of the increase in the relative volatility of wages when γ grows comes from the labour supply curve. A higher value of γ makes the labour supply curve steeper, and this implies larger volatility of wages but lower volatility of hours.

A higher level of θ should imply a decrease in the volatility of consumption. However, Table 2.7 reports volatilities with respect to output, and not in absolute terms. When θ is higher households are less eager to substitute consumption over time, and as a result their labour supply moves less after a shock[28]. This implies that the volatility of hours decreases, while the volatility of wages increases. Moreover, output follows closely the shock, with lower amplification due to labour adjustment, and, by equation (2.3), consumption follows more closely wages and therefore output. Since the labour supply moves less after a shock when θ is higher, hours have higher first-order autocorrelation and lower correlation with

[27] Consumption smoothing and "leisure smoothing" depend on two separate parameters, but also on the relative importance of one parameter with respect to the other.

[28] Equation (2.3) gives the supply of hours of labour h_t as a function of the real wage w_t.

output.

Figures 1 to 3 show the responses of output, hours, consumption and wages to a 1% positive technology shock. Responses are plotted for different values of γ, while θ stays fixed at 1. It is possible to see that different values of γ do not affect or affect only slightly the responses of consumption and wages. For $\theta = 2$ the responses (not shown) of output, consumption and wages are dampened, and the impact of the shock on hours becomes very small.

As it is known already, the indivisible labour model is the one that delivers the maximum contemporaneous impact of the shock on labour and output. However the above discussion about relative volatilities suggests that the impact on hours depends also on the relative importance of the intertemporal substitutability of consumption, with respect to the substitutability of hours.

Results

Values of θ between 1 and 2 are the most commonly used in the literature, and in practice there is some degree of freedom in the choice of the intertemporal elasticity of consumption. However, when γ is equal to zero all the fluctuation in the labour input can be attributed to employment instead of hours. This has the advantage of abstracting from hours. Therefore, with $\gamma = 0$ the parameter θ is chosen so as to generate an exact match of the employment volatility in the data.

Table 2.8 reports the second order moments generated by the model with θ equal to 1.33 and γ equal to zero, and all the other parameters chosen as described above. It can be seen that, having chosen θ so as to match the employment volatility, the model generates a relative standard deviations for investment close to the one in the data. All the variables are procyclical, and first-order autocorrelations are satisfactory for output, consumption, investment and employment[29]. Cross-correlations with output are satisfactory for consumption and investment. However, the model generates too low volatility for all the variables except employment and investment, and too high correlations with output for employment and wages.

The ability of this RBC model to match real-world statistics may be limited by the

[29] The value for ρ was chosen in order to reproduce as closely as possible the first-order autocorrelation of output. However, even with the very high value $\rho = 0.99$, the first-order autocorrelation of output is lower than in the data.

Table 2.8: Standard deviations and correlations with output

$\theta = 1.33$, $\gamma = 0$	St. dev. rel. to output	1-st ord AC	Cross-correlation of output at time t with x(t-1)	x(t)	x(t+1)
Output	1.00	0.71	0.71	1.00	0.71
Consumption	0.33	0.72	0.65	0.97	0.74
Investment	4.39	0.70	0.71	0.99	0.68
Employment	0.56	0.70	0.73	0.99	0.66
Wages	0.46	0.72	0.66	0.98	0.74
Interest rate	0.02	0.70	0.73	0.99	0.66

assumption that there is only one source of fluctuation, the shock to technology. It is therefore interesting to see what happens if the stochastic structure of the model is enriched by introducing another shock. In order to try to do so, in the next section Italy is modelled as a small open economy, where uncertainty comes from both a technology shock and a shock in the world rate of return on bonds.

2.4 The Open Economy Model

The model presented in this section is the extension to the open economy of the same closed economy model of the preceding section. There is a single asset, an internationally traded bond, the rate of return on which is exogenous. Models of this sort are used to model small open economies, because the return on the asset cannot be influenced endogenously. It's reasonable to assume that Italy is a small, open country, since the ability to control the interest rate has been limited by the exchange rate mechanism first, and more recently by the introduction of the single currency[30].

The model

The same assumptions on the agents, the productive sector, and the institutional setting continue to hold, but we need two more assumptions: the first is that labour is internationally immobile, and the second is that there are adjustment costs for the stock of capital. Moreover, whenever the agents' holdings of the internationally traded bond are

[30] Italy had been a member of the ERM since its inception in 1978, first with a fluctuation band of $\pm 6\%$, and then, from 1990 to 1993, with a fluctuation band of $\pm 2.25\%$. The European Single Currency aws introduced in January 1999.

different from the long run or steady state level, agents face a portfolio adjustment cost. In this way, the equilibrium dynamics is stationary and the log-linearization procedure is consistent[31].

As in the previous section, households maximise their expected utility over infinite sequences of consumption $\{c_t\}_{t=1}^{\infty}$ and leisure $\{l_t\}_{t=1}^{\infty}$ pairs:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] .$$

Households can consume more or less of what they earn in each period because they can sell or buy bonds in the international capital market at the exogenous real interest rate r. Their budget constraint satisfies:

$$c_t + i_t + \Phi\left(k_{t+1} - k_t\right) + \frac{\psi}{2}\left(b_{t+1} - \overline{b}\right)^2 + b_{t+1} = z_t k_t^{1-\alpha} h_t^{\alpha} + b_t\left(1 + r_t\right) ,$$

where b is the stock of traded bonds, r is the return on bonds, $\Phi(\cdot)$ is a convex function and ψ is a constant parameter of the portfolio adjustment cost function. The total amount of time available to the agents is normalised to 1. Adjustment costs for capital are introduced in order to prevent the equalisation in every period of the rate of return on physical capital and the return on bonds, by arbitrage. The function $\Phi(\cdot)$ is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$, so that in the steady state there are no capital adjustment costs.

As before, the production function is Cobb-Douglas with constant returns to scale, and firms and households act as price-takers. Only one good is produced, which can be consumed, transformed into capital or traded for foreign assets. The international capital market is therefore another channel (together with capital accumulation) to smooth consumption over time. The equilibrium rental prices for labour and capital are given as before by (2.1) and (2.2). The law of motion for capital is:

$$k_{t+1} = (1-\delta)k_t + i_t .$$

The log-deviations of the exogenous process for technology z_t and the return on bond r_t are assumed to follow an AR(1) process:

$$\widehat{z}_{t+1} = \rho_z \widehat{z}_t + \varepsilon_{t+1}^{z} ,$$

[31]See Schmitt-Grohe and Uribe (2003).

$$\widehat{r}_{t+1} = \rho_r \widehat{r}_t + \varepsilon^r_{t+1} \ ,$$

with the shocks ε^z_t i.i.d $N\left(0, \sigma^2_z\right)$ and ε^r_t i.i.d $N\left(0, \sigma^2_r\right)$. Shocks to technology and the exogenous interest rate have only temporary effects. It is therefore assumed that short-run fluctuations can be modelled independently from long-run growth.

The current account of this economy, which is savings minus investments, plus net interest payments from the international bond, is given by:

$$CA_t = b_{t+1} - b_t \ ,$$

which is equal to minus the capital account of the economy. The variable b_t is the amount of credit of the domestic economy at time t.

The optimality conditions of the household maximisation problem are:

$$c_t^{-\theta} w_t = A\left(1 - h_t\right)^{-\gamma} \ , \tag{2.6}$$

$$\beta\, E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\theta}\left(1 + r_{t+1}\right)\right] = E_t\left[1 + \psi\left(b_{t+1} - \overline{b}\right)\right] \ , \tag{2.7}$$

$$c_t^{-\theta} E_t\left[1 + \Phi'\left(k_{t+1} - k_t\right)\right] = \beta\, E_t\left\{c_{t+1}^{-\theta}\left[r^k_{t+1} + 1 - \delta + \Phi'\left(k_{t+2} - k_{t+1}\right)\right]\right\} \ . \tag{2.8}$$

The first condition is an intratemporal efficiency condition, relating hours of work with consumption choices, as in Section 2.3. The other two conditions are arbitrage conditions, relating the return on physical capital and the international bond to the optimal intertemporal allocation of consumption.

Calibration

The performance of the model for a few significant values of θ and γ is checked, as in Section 2.3, and where possible, parameters are calibrated at the same values given in Section 2.3.

For example, the parameters δ and α are set at the same values used in the closed economy model. The calibration of A is done as in the closed economy, once the parameters

Table 2.9: Parameter and steady state values

δ	α	h	β	$\frac{b}{y}r$	ψ	ρ_z	ρ_r	σ_r
0.0088	0.33	0.12	0.99	-0.005	0.028	0.99	0.37	0.006

θ and γ are decided, so as to ensure that in the steady state h is equal to 0.12. The equation is modified to take into account the steady-state share of the current account on output:

$$A = (1-h)^{\gamma}\left(1+\frac{b}{y}r-\delta\frac{k}{y}\right)^{-\theta}(1-\alpha)\,h^{-\theta}\left(\frac{y}{h}\right)^{(1-\theta)} .$$

The extension to the open economy requires the calibration of some additional parameters and steady state values. The average of the ratio current account/output in Italy for the period 1970(I) to 1998(III) is -0.005. This is the value used in the calibration of the steady state level of the internationally traded bond. The parameter ψ is taken from Schmitt-Grohe and Uribe (2003), while the capital adjustment cost function $\Phi(\cdot)$ does not enter the log-linearized equations, therefore its calibration is not necessary.

The autoregressive process for the interest rate is estimated using data on short-term real rates in Germany, given that this is the country whose policy is more likely to affect Italian rates of return on bonds. The discount factor β is calibrated so that $\beta(1+r)=1$, given the steady state or average value of the rate of return on bonds, so it is set at the same value as in the closed economy. The variance of the innovations in the process for technology is again chosen so as to match the volatility of output in the data, while the autoregressive coefficient is set equal to 0.99.

To summarise this Section, Table 2.9 reports the calibrated parameter and steady state values.

Short-run dynamics

Figures 4 to 9 show the response of the main economic variables to 1% positive shocks in technology and the interest rate. However, before analysing the responses and explaining the model results, it is possible to predict, intuitively, the main consequences of the openness assumption. Since households have an additional channel to smooth consumption over time, the volatility of consumption will probably be lower than in the closed economy. Furthermore, to take fully advantage of the productivity gains, households will increase their labour supply more after a positive technology shock, knowing that they can enjoy the associated output surge also in the future via lending. Therefore, the volatility of hours will increase in the open economy. Finally, the presence of an additional endogenous predetermined variable (b_t) will increase the persistence of the other variables.

Now, in order to illustrate the short-run dynamics, let us consider the technology shock first. The technology shock increases output and wages, consumption and investment. Because of the convex portfolio adjustment costs, agents do not fully use the international bonds market to smooth their consumption in the first period, therefore consumption continues to grow, even if the discount factor β and the real interest rate r are at the same level as before. The indivisible labour model amplifies the response of hours and output to the technology shock. Responses are not plotted for a different intertemporal elasticity of consumption. This is because $\theta = 2$ does not affect the shape and magnitude of the responses.

Now let us consider the interest rate shock. If households have a negative b, the shock in the interest rate increases the cost of the debt, therefore households reduce their bond holdings. If they had a positive b, an increase in bond holdings would take place. Physical investment decreases because foreign assets are more rewarding than physical capital.

The impulse response of investment to both the technology and the interest rate shocks is kinked because both shocks drive a wedge between the expected and the realized return on capital and the international bond, respectively. The kink in the response of investments arises because the conditional expectation of the shock is equal to 0 in period $t = 1$, but from $t = 2$ onwards the shock is correctly predicted. So, when $t = 2$ investment suddenly changes to re-equalise the returns on capital and the international bonds (minus adjustment costs). From that moment onwards agents do not need to revise their expectations and the response of investment is gradual. Because of the sudden change in the response of investment in period $t = 2$, output, wages and hours also display a kink in the response to the shocks.

Results

Instead of assessing the model performance under different values for the intertemporal elasticities of substitution, it is perhaps more interesting to compare the results with the closed economy, for a given parametrization. I compare the results for the two economies

Table 2.10: Open economy: Standard deviations and correlations with output

$\theta = 1$, $\gamma = 0.5$	St. dev. rel. to output	1-st ord AC	Cross-correlation of output at time t with x(t-1)	x(t)	x(t+1)
Output	1.00	0.74	0.74	1	0.74
Consumption	0.34	0.74	0.68	0.98	0.76
Investment	6.99	0.19	0.63	0.78	0.16
Hours	0.63	0.75	0.76	1	0.72

Table 2.11: Closed economy: Standard deviations and correlations with output

$\theta = 1$, $\gamma = 0.5$	St. dev. rel. to output	1-st ord AC	Cross-correlation of output at time t with x(t-1)	x(t)	x(t+1)
Output	1.00	0.71	0.71	1.00	0.71
Consumption	0.33	0.73	0.64	0.97	0.74
Investment	4.42	0.70	0.71	0.99	0.67
Hours	0.62	0.70	0.72	0.99	0.67

when $\theta = 1$ and $\gamma = 0.5$. The effect of different elasticities is quite predictable: when $\theta = 2$ the volatility of consumption goes down, and because households are less eager to substitute consumption over time, their labour supply moves less after a shock, and the volatility of hours is diminished (see Section 2.3 and the footnote on page 43). When γ is equal to 1 labour supply is steeper and the volatility of hours is lower, therefore $\gamma = 0.5$ gives a better performance in terms of the volatility of hours. Values of γ below 0.5 would increase the volatility of hours, however, on the one hand the high intertemporal elasticity of substitution of leisure may not be confirmed by the empirical research, and on the other hand it is interesting in itself to see whether it is possible to increase the volatility of hours by the openness assumption.

Tables 2.10 and 2.11 report the second moments for the open and the closed economy. As it can be seen from the tables, openness does not necessarily reduce the volatility of consumption. This unexpected result can be explained by the presence of portfolio adjustment costs, since they reduce the agents' ability to smooth consumption through the international bonds market. As a result, the volatility of consumption does not always fall in the open economy. Moreover, the sudden change in the response of investment to both shocks explains the high volatility and the low autocorrelation of investments.

There is a moderate increase in the volatility of hours in the open economy. This can be explained by the availability of an additional channel to smooth consumption over time. After a technology shock, households can enjoy more consumption today by means of temporarily increasing the foreign debt, which will be brought back to the equilibrium level by means of future higher output. Since a high level of output is required in the future, the responses of hours and investment are quite strong. Therefore, in the open economy the volatility of hours is increased, but at the expense of an increase in the volatility of investments.

In conclusion, the analysis shows that the open economy model is mildly successful in generating an increased volatility of hours without the need to assume that the intertemporal elasticity of substitution of leisure is large, which may not be supported by microeconometric evidence. However, with respect to investment, the open economy model generates a too high volatility.

As was stressed before, these results depend on the parametrization chosen and on the form of the utility function. It is generally true, however, that the introduction of more than one shock can improve the performance of a RBC model along many dimensions.

In spite of these successes, there are also some drawbacks. First, the model still was not able to replicate the very large volatility of hours observed in the data. Second, the increased volatility of hours was obtained at the cost of an increase in the volatility of investment to a level that is too high to match the data.

2.5 Conclusion

This chapter applies several RBC models to the Italian economy to see whether they can explain the aggregate fluctuations observed in the data. The performance of these models depends crucially on the parametrization chosen and on the form of the utility function, in particular, the intertemporal elasticities of substitution of labour and consumption both affect the short-run dynamics and the volatilities generated by the model. The chapter also analyses the introduction of more than one shock, considering Italy as a small open economy subject to exogenous interest rate shocks.

The message of this chapter is the following. The volatility of hours can be increased by assuming a high elasticity of intertemporal substitution of leisure, or by introducing the possibility of borrowing and lending from the rest of the world. However, results from the simulations have shown that the increase in the volatility of hours either happens at the expense of the volatility of consumption, which becomes too low, or at the expense of the volatility of investment, which becomes too high. This phenomenon can be explained by recalling that in the RBC model the motive for changing the labour supply is intertemporal substitution. For example, a positive technology shock increases the payoff of working longer hours, but agents want to spread the payoff into the future via capital accumulation. Hence, consumption moves less than one for one with output, and investment increases. Moreover, an increase in labour supply elasticity results in more consumption smoothing, thus lowering the volatility of consumption or increasing the volatility of investment.

This helps to explain why the standard RBC model cannot reproduce the stylized facts of the Italian labour market, namely that hours fluctuate much more than employment. The possibility of increasing the responsiveness of labour supply by changing the model parameters (beyond the values implied by microeconometric evidence for other countries) is limited by the consideration that if this strategy had been tried either the volatility of consumption would have become too low to match the data, or the volatility of investments would become too high.

However, the standard RBC model was "designed" for the US, therefore its inability to reproduce the volatility of hours in Italy may not be inherently due to its propagation mechanism, but rather to the neglect of some specific labour market institutions or features of the Italian economy.

In particular, the Italian labour market is more regulated than the US one, and as a consequence it is costlier for firms to adjust the number of employees during recessions. The high volatility of hours in Italy may be plausibly attributed to rigidities in the labour market, which prevent employers from firing after a bad shock. These rigidities may also be the cause of the lower standard deviation of employment in Italy, compared to the US. To evaluate whether a RBC model can capture the high volatility of hours, the next chapter introduces a model with both hiring and firing costs, as a way to model labour market rigidity.

Moreover, in both the open and the closed economy models of this chapter the household's decision to supply labour had only one dimension, hours, or in the case of the indivisible labour model, participation only. However, to reproduce both the volatility of hours and employment in the Italian economy it is necessary to have a model where households can make a decision on both their labour market participation and the hours of work. Therefore, all the models presented in the next chapter distinguish between these two dimensions of the labour supply.

Appendix 2.A: Definitions and sources

Table 2.12: Data

Series	Definition	Source of data used in calculations
GDP	Gross Domestic Product at current prices / GDP implicit price deflator	OECD Quarterly National Accounts
Capital Stock	Calculated by the following iteration: Net Stock (t) = Net Stock (t-1) + Total investment - (Capital Consumption / GDP implicit price deflator). Capital Consumption for Italy is annual data / 4.	Net Stock: OECD Flows and Stocks of Fixed Capital. Other: OECD Quarterly National Accounts
Employment	USA: Total Labour Force - Total Unemployment	Italy: Bank of Italy USA: OECD Quarterly Labour Force Statistics
Per capita hours	Italy: monthly hours of blue-collar workers in industrial firms, de-indexed, adjusted for seasonality. Quarter averages of monthly data. USA: average weekly hours of production workers. Quarter averages of monthly data.	Italy: ISTAT USA: Bureau of Labor Statistics
Wages per capita	Adjusted Compensation of Employees / Employment Adjusted Compensation of Employees = (Compensation of Employees / Consumer Price Index) * Employment / Employees	OECD Quarterly Labour Force Statistics and Quarterly National Accounts
Wages per hour	Adjusted Compensation of Employees / (Employment * Per capita hours)	
Output per hour	GDP / Employment	OECD Quarterly Labour Force Statistics and Quarterly National Accounts
Output per capita	GDP / (Employment * Per capita hours)	

Table 2.13: (continues) Data

Series	Definition	Source of data used in calculations
Private consumption	Private Final Consumption Expenditure at current prices / Private Final Consumption implicit price deflator	OECD Quarterly National Accounts
Government cons	Government Final Consumption Expenditure at current prices / Government Final Cons. implicit price deflator	OECD Quarterly National Accounts
Total consumption	Private cons. + Government cons.	
Fixed investment	Gross Fixed Capital Formation / GFCF implicit price deflator	OECD Quarterly National Accounts
Inventory inv. / GDP	(Increase in Stocks / GFCF implicit price deflator) / GDP	OECD Quarterly National Accounts
Total investment	(Gross Fixed Capital Formation + Increase in Stocks) / GFCF implicit price deflator	OECD Quarterly National Accounts
Current account / GDP	(Exports- Imports) / GDP	
Exports	Exports of Goods and Services at current prices / Exports implicit price deflator	OECD Quarterly National Accounts
Imports	Imports of Goods and Services at current prices / Imports implicit price deflator	OECD Quarterly National Accounts

Chapter 3

Real Business Cycle Theory and the Volatility of Hours in Italy

3.1 Introduction

The analysis of the previous chapter has shown that, in order to be able to reproduce the cyclical fluctuations of the Italian economy, the standard RBC model needs to be modified in some way. In particular, this chapter considers the possibility of modelling explicitly two institutions or features that characterise the Italian economy: a high degree of labour market rigidity, and the existence of a large underground economy. Obviously, these are not the only two characteristics that set Italy apart from the US or other countries[1], but for the purpose of "approximating reality" they might still do a good job. In fact, both the official statistics and a range of independent studies rank Italy among the countries with the highest degree of labour market rigidity, and the US the lowest; Italy is also the OECD country with the largest underground sector[2]. More interestingly, the presence of labour market rigidities and the sizeable underground sector can explain why in Italy hours of work fluctuate more than employment. Rigidities prevent or make it costlier for registered, "official" firms to change the number of employed workers, and as a result they prefer to change hours instead of employees in response to shocks. Individuals can offset the change in the registered or "official" hours by shifting from the production of goods in the registered sector to the production of goods in the unregistered sector, or vice versa.

[1] As seen in the previous chapter, Maffezzoli (2001) considers the introduction of trade unions.
[2] Figures will be given in Sections 3.2 and 3.3.

In this chapter the specific type or source of labour market rigidity are the adjustment costs of the labour force. Firms have to pay the adjustment cost both for hiring and firing workers. It is important to point out that, while firing and hiring costs are not the only source of labour rigidity, they nevertheless can be thought of as a simple but flexible modelling tool, capable of accommodating several arrangements. In fact, firing and hiring costs can either be due to legislation, or they may be generated by market inefficiencies. For example, the hiring cost can also be interpreted as a search cost.

There are several RBC models that include labour adjustment costs or a nonmarket sector, but not with the purpose of increasing the volatility of hours worked. The models presented in this chapter have several features in common with the model of Kydland and Prescott (1991): both hours of work and number of employees are choice variables, and there is a cost of moving in and out of work. Kydland and Prescott find in fact that moving costs increase the standard deviation of hours per worker, which nevertheless never reaches the value of double the standard deviation of output (as it would be needed to match the Italian data), even when the moving costs are very large. However, in Kydland and Prescott's model contemporaneous utility is nonseparable, therefore the intertemporal elasticity of substitution of leisure cannot be changed independently. It is therefore interesting to see the performance of the RBC model with moving costs under different values for the intertemporal elasticity of substitution of leisure, which is what Section 3.2 does.

There are also several RBC models that include a non-registered, "household" sector, as opposed to a "market" sector, for example Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). Although they do not introduce a household sector to explicitly increase the volatility of hours, the results from their simulations are conclusive in showing that in the RBC model with a household sector hours worked in the market sector have higher standard deviation. One limitation of Benhabib, Rogerson, and Wright, and Greenwood and Hercowitz' approach is that, because of their assumption that the good produced in the household sector cannot be traded, their models do not consider all other types of unregistered activities that result in the sale of goods. In order to verify the hypothesis that Italy's large underground economy is responsible for the cyclical behaviour of the labour market, it seems therefore necessary to modify the standard RBC model by including one additional sector, while removing at the same time the assumption of nontradability. This is what Section 3.3 does.

The results from the simulations of both models are only in part satisfactory. The introduction of firing and hiring costs has the desired effect on the volatility of hours, but it is not possible to match closely the high value observed in the Italian data. However, the model with the underground sector performs better with respect to the other variables, and it is able to generate a relatively high volatility of hours with small adjustment costs.

This chapter is organised as follows. Section 3.2 introduces hiring and firing costs in the model of Section 2.3, and Section 3.3 further adds an underground sector. Finally, Section 3.4 concludes.

3.2 The Model with Firing Costs

In this chapter both firing and hiring costs are introduced, in order to see whether the model of the previous chapter can generate the high volatility of hours worked and the comparatively lower volatility of employment that characterise the Italian data. It is however difficult to assess the size of hiring and firing costs or labour market rigidity in Italy comparatively to other countries. One difficulty is that there are many ways to prevent or increase the cost for firms of hiring and firing[3].

The OECD's EPL (employment protection legislation) index is a comprehensive measure that refers both to legislated and *de facto* regulations concerning hiring and firing[4]. According to the OECD's Employment Outlook, Italy is ranked twenty-third for its degree of labour market rigidity in the late 1990s, having a score of 3.4 in overall EPL strictness[5]. Thus the OECD places Italy very close to Portugal, the country with the highest degree of labour market rigidity, which is ranked twenty-sixth, and very far from the US, which is ranked first[6]. This view is confirmed by other studies. For example, the same OECD publication shows various other rankings that have been used by earlier studies to compare the strictness of EPL across countries. These rankings were published by the International Organisation of Employers (1985), the EC ad hoc surveys (1991, 1995), Lazear (1990), Bertola (1990) and the OECD Jobs Study (1994), and all of them place Italy among the

[3]Quoting from Schivardi and Torrini (2004): "Firing costs can be thought of as the result of three main elements: the definition of fair and unfair dismissal; the cost of a no-fault dismissal and the penalty when the dismissal is ruled to be unfair; the uncertainty on the result of a possible trial".

[4]It includes administrative procedures for individual notice and dismissal; required notice and severance pay; conditions under which individual dismissals are fair or unfair; regulation of fixed-term contracts and of temporary work agencies.

[5]The overall EPL strictness index runs from 0 to 6.

[6]Source: OECD (1999), Chapter 2.

countries with the highest degree of EPL strictness.

One of the consequences of the labour market rigidity in Italy is that a lot of labour adjustment takes place through the social security system. In fact, under some conditions, a troubled firm can have access to the so-called "Cassa Integrazione Guadagni", which allows it not to employ some or all of its workers, without having to dismiss them, at least for a short period of time. Workers do not lose their job, and they receive a payment from the social security system that covers almost entirely their wage. What is important is that they do not become unemployed, so this sort of labour adjustment is, as far as statistics are concerned, variation in hours and not "bodies". It is also plausible that the same sort of restrictions which prevent the firms from firing work also in the opposite direction, making firms more reluctant to hire during expansions. If this is the case, a lot of labour adjustment will take place during expansions by resorting to overtime, instead of recruiting new workers. This again causes a change in hours worked, while employment is only mildly affected or not affected at all.

The model

The models of the previous sections did not make a distinction between the decision to participate or not in the labour market, and the decision on how many hours to supply. In reality, labour fluctuates along both the intensive margin and the extensive margin, a feature that can be captured only in a model where the two dimensions of variation of the labour input are explicitly modelled[7].

There is a continuum of agents on the interval [0,1]. The utility function of each agent is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] ,$$

where c_t is consumption and l_t is leisure. The time endowment of each individual is normalised to 1, but if the individual decides to work he loses for commuting an amount of time equal to $1 - \psi$, where $\psi \in (0,1]$ is a fixed parameter. Consequently, the amount of leisure that the individual enjoys is equal to one if he does not work, and if he works it is equal to $\psi - h_t$, where h_t are hours worked. The parameter ψ models the cost of participation in the labour market.

[7]The modelling strategy used in this section follows quite closely Kydland and Prescott (1991).

Firms produce a composite good, which is given by the number of hours times a constant returns to scale production technology:

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t \ ,$$

where y_t is output, z_t is the technology shock, k_t is capital and n_t is employment, so unemployment is $1 - n_t$. The adjustment costs of the labour force are quadratic, and they are paid by the firms whenever they lay off workers or recruit new ones:

$$AC_t = \frac{\phi}{2} \left(n_t - n_{t-1} \right)^2 \ .$$

The law of motion for capital is:

$$k_{t+1} = \left(1 - \delta \right) k_t + i_t \ ,$$

where δ is the depreciation rate. Capital at time zero, k_0, is given, as well as employment at time $t = -1$, n_{-1}. The logarithm of the technology shock follows an AR(1) process.

Unemployment is the only source of heterogeneity, since agents are the same in this economy. As a consequence, all the unemployed have the same level of consumption, and all the employed work the same number of hours and consume the same amount of the composite good. Individuals choose employment lotteries and either there is a market where individuals can insure themselves, or employment insurance is provided by firms.

As shown by Appendix 3.A, if both lotteries and a market for contingent contracts, which enables agents to insure income risk, are introduced in the model economy, then the competitive equilibrium is Pareto-optimal. As a result, it is possible to find the solution path by solving the following social planning problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ (1 - n_t) \left[\frac{(c_t^u)^{1-\theta} - 1}{1 - \theta} \right] + n_t \left[\frac{(c_t^e)^{1-\theta} - 1}{1 - \theta} + A \frac{(\psi - h_t)^{1-\gamma} - 1}{1 - \gamma} \right] \right\} \ ,$$

subject to:

$$c_t + i_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t - \frac{\phi}{2} \left(n_t - n_{t-1} \right)^2 \ .$$

$$c_t = (1 - n_t)\, c_t^u + n_t c_t^e \ ,$$

$$k_{t+1} = (1 - \delta)\, k_t + i_t \ ,$$

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1} \ ,$$

where c_t^u is consumption of the unemployed and c_t^e is consumption of the employed. ε_t i.i.d $N\left(0, \sigma^2\right)$. It follows that optimality requires:

$$A\, (\psi - h_t)^{-\gamma} = c_t^{-\theta} z_t k_t^{\alpha} n_t^{-\alpha} \ , \tag{3.1}$$

$$c_t^{-\theta} = \beta\, E_t \left[c_{t+1}^{-\theta} \left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta\right)\right] \ , \tag{3.2}$$

$$A \frac{(\psi - h_t)^{1-\gamma} - 1}{1 - \gamma} + c_t^{-\theta} (1 - \alpha)\, z_t k_t^{\alpha} n_t^{-\alpha} h_t = c_t^{-\theta} \phi\, (n_t - n_{t-1}) - \beta\, E_t \left[c_{t+1}^{-\theta} \phi\, (n_{t+1} - n_t)\right] \ . \tag{3.3}$$

Because preferences are separable in leisure and consumption, $c_t^u = c_t^e = c_t$. Equations (3.1) and (3.2) are the familiar intratemporal and intertemporal efficiency conditions. Equation (3.3) describes the optimal allocation of agents between employment and unemployment. The left-hand side is the marginal benefit of adding an additional worker to production, and the right-hand side is the present discounted value of the marginal cost.

Calibration

This economy has a unique nonstochastic steady state in which the shock is equal to its mean value and the variance of innovations is zero. The parameter ψ is set so as to match a given level of hours worked and the parameter A is set so as to match a given participation rate in the steady state. In fact, dropping time subscripts, in the steady state the following must be true:

$$\frac{(\psi - h)^{1-\gamma} - 1}{1 - \gamma} = -\,(1 - \alpha)\, (\psi - h)^{-\gamma}\, h \ .$$

Table 3.1: Parameter values

δ	α	β	h	n	ρ
0.0088	0.33	0.99	0.32	0.37	0.99

The above equation can be solved for ψ, once α and h have been calibrated and a choice of γ has been made.

Equation (3.2) gives the capital-output ratio in the steady state:

$$\frac{y}{k} = \frac{1}{\alpha}\left(\frac{1}{\beta} + \delta - 1\right) ,$$

from which the output-employment ratio $\frac{y}{n}$ can be derived. Then a consistent measure of the parameter A is given by the following:

$$A = (\psi - h)^{\gamma}\left(1 - \delta\frac{k}{y}\right)^{-\theta}\left(\frac{y}{n}\right)^{1-\theta} n^{-\theta}h^{-1} ,$$

which shows that n, the fraction of population that works, must also be calibrated at some sensible level. Therefore n is set equal to the average employment -population ratio.

The fraction of time h devoted to market activities by working people is given by the average hours worked in a week (35.53), divided by total time available (see footnote on page 40).

The parameters θ, γ, and ϕ are set to a benchmark parametrization, and then some sensitivity analysis is performed.

The parameters δ, α, β, and ρ are set at the same values used previously. As before the variance of innovations of the process for technology is set at the level that reproduces the standard deviation of output in the data. The values of the parameters are reported in Table 3.1.

Results

Table 3.2 reports the standard deviations of employment, hours, consumption and investment, relative to the standard deviation of output, obtained by simulating the model for different values of the adjustment cost. Since the feature of the data to be reproduced is the high standard deviation of hours, only the value $\gamma = 0$, or infinite intertemporal elasticity of substitution of hours, is considered.

Table 3.2: Model results

	Relative standard deviation			
	n	h	c	i
$\theta = 2, \gamma = 0$				
$\phi = 0$	0.37	0.00	0.33	4.19
$\phi = 1$	0.43	0.54	0.34	4.34
$\phi = 1.5$	0.40	0.82	0.29	4.82
$\phi = 2$	0.33	1.00	0.22	5.29
$\phi = 2.5$	0.25	1.07	0.15	5.56
$\phi = 3$	0.17	1.08	0.09	5.67
$\theta = 1, \gamma = 0$				
$\phi = 0$	0.68	0.00	0.33	4.20
$\phi = 1$	0.68	0.85	0.28	4.87
$\phi = 1.5$	0.51	1.05	0.19	5.39
$\phi = 2$	0.36	1.10	0.12	5.60
$\phi = 2.5$	0.25	1.10	0.07	5.67
$\phi = 3$	0.17	1.09	0.04	5.68

When $\phi = 0$ there are no labour adjustment costs, and, since the instantaneous utility of leisure hours is a convex function, households change their labour supply after a shock by changing n_t, while hours h_t do not move. In this case, as Table 3.2 shows, the standard deviation of consumption is exactly the same as in the model of Section 2.3, and the standard deviation of employment n_t is equal to the one of hours in the same model. This shows that when $\phi = 0$ this model becomes equivalent to the model of Section 2.3, except that that n_t is substituted for h_t.

As Table 3.2 shows, an increase in the adjustment cost parameter ϕ is successful in increasing the relative volatility of hours with respect to employment. ϕ takes values between 1 and 3, which imply that the marginal cost of a 1-percent change in the employment level is between 0.002 and 0.007 percent of a quarter's output. These values are well below the level employed by Cogley and Nason (1995), who proposed an estimate for the marginal cost equal to 0.36 percent. Higher values for ϕ introduced complex roots in the model.

When ϕ increases households become more willing to substitute hours over time, in order to avoid the hiring and firing costs. As a result, they become comparatively less willing to substitute consumption over time, and the relative volatility of consumption decreases, as Table 3.2 shows.

As in the model of Section 2.3, an increase in θ tends to induce a decrease in the volatility of consumption relative to output. As explained before, this happens because when θ is higher households are less eager to substitute consumption over time, and as a result their labour supply moves less after a shock. This implies that the volatility of hours decreases. When the intertemporal elasticity of substitution of leisure γ and ϕ are equal to zero, all the adjustment is made through employment.

The increase in the relative volatility of hours is accompanied by a decrease of the relative volatility of employment, which is an unappealing feature of this model. The failure to reproduce relative standard deviations close to those in the data is a consequence of the short-run dynamics induced by the model. As it can be seen from Figure 10, the labour adjustment cost has an impact on hours in the first quarter only, then hours stay constant afterwards, which implies that the adjustment cost does not "bite" any longer. This happens because households adjust hours only in the first quarter because n_{t-1} is given, then they find paths for employment and consumption such that the left-hand side of Equation (3.3) (the only dynamic equation with labour adjustment costs) is equal to zero. But then equations (3.1) and (3.3) together imply that hours are always constant if $\phi = 0$, otherwise they adjust only in the first period. Since hours adjust only once, ϕ must be high to generate a high volatility of hours, but then the standard deviations of employment and consumption become too low, and the standard deviation of investment becomes too high. Also, this model generates a negative correlation of hours with employment, while in the data it is positive and close to zero.

These problems can be overcome by the choice of nonseparable preferences. When preferences are nonseparable, hours do not enter the optimality conditions separately from consumption, therefore, they must follow a smooth path, and not being adjusted one period only. It will not be possible to find paths for employment and consumption such that the left-hand side of the optimality condition (3.3) is always zero. Assuming that the utility function of each agent is:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^{\mu} \, l_t^{1-\mu} \right)^{1-\theta} - 1}{1-\theta} \right] ,$$

the optimality conditions become:

Table 3.3: Model results with nonseparable preferences

	Relative standard deviation			
	n	h	c	i
$\theta = 2, \gamma = 0$				
$\phi = 0$	0.48	0.02	0.58	2.65
$\phi = 1$	0.27	0.13	0.58	2.65
$\phi = 1.5$	0.24	0.18	0.59	2.64
$\phi = 2$	0.22	0.19	0.59	2.64
$\phi = 2.5$	0.20	0.20	0.60	2.63
$\phi = 3$	0.18	0.21	0.60	2.63

$$(c_t^u)^{\mu-\mu\theta-1} = (c_t^e)^{\mu-\mu\theta-1} (\psi - h_t)^{(1-\mu)(1-\theta)} ,$$

$$(c_t^u)^{\mu-\mu\theta-1} = \beta E_t \left[\left(c_{t+1}^u\right)^{\mu-\mu\theta-1} \left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta\right) \right] ,$$

$$\begin{aligned} &\frac{(c_t^u)^{\mu(1-\theta)} - 1}{1-\theta} - \frac{\left((c_t^e)^{\mu} (\psi - h_t)^{1-\mu}\right)^{1-\theta} - 1}{1-\theta} - \mu \left(c_t^u\right)^{\mu-\mu\theta} + \\ &\quad + \mu \left(c_t^e\right)^{\mu-\mu\theta} (\psi - h_t)^{(1-\mu)(1-\theta)} = \\ &= \mu \left(c_t^u\right)^{\mu-\mu\theta-1} (1-\alpha) z_t k_{t-1}^{\alpha} n_t^{-\alpha} h_t - \mu \left(c_t^u\right)^{\mu-\mu\theta-1} \phi \left(n_t - n_{t-1}\right) + \\ &\quad + \beta E_t \left[\mu \left(c_{t+1}^u\right)^{\mu-\mu\theta-1} \phi \left(n_{t+1} - n_t\right) \right] . \end{aligned}$$

$$z_t k_{t-1}^{\alpha} n_t^{-\alpha} = \frac{1-\mu}{\mu} \frac{c_t^e}{\psi - h_t}$$

Table 3.3 reports the results for the model with nonseparable preferences. Parameter values are calibrated as before, μ is calibrated so as to match the employment level in steady state, and the case $\theta = 1$ is excluded because it introduced complex roots in the approximated solutions. The model in unsuccessful in generating a standard volatility of hours much higher than employment. As with separable preferences, the increase in the volatility of hours takes place at the expense of the volatility of employment.

3.3 The underground economy and firing costs

One of the characteristic features of Italy is the existence of a large underground economy. The estimated size of the Italian underground sector in 1998 was, according to Istat, between 14.7 and 15.4% of GDP, while, according to the European Commission, the average size of the underground economy within the European Union is between 7 and 16% of GDP. However, these estimates from Istat are regarded as "prudent". For example, Schneider and Enste (2000; 2002) estimated the size of the Italian underground sector in 1994 to be 27.8% of GDP, the highest percentage among all OECD countries, and also growing over time[8].

These numbers suggest that the existence of an alternative to registered market activities may provide an explanation for the high volatility of hours observed in the Italian data. The assumption is that time spent in registered activities and time spent in unregistered activities do not enter the household's utility function separately, as households have preferences over consumption and total hours worked, irrespective of the sector in which they work. Then, households in Italy may be willing to accept a high volatility of hours in the registered sector because, by switching from the market sector to the underground economy and vice versa, they can moderate the volatility of total hours worked, which affects their lifetime utility.

It must be pointed out, however, that the notion of an "unregistered" or "non-observed"[9] economy does not coincide with the notion of an underground economy, and it seems necessary at this point to draw some distinctions. According to Blades and Roberts (2002), the group of non-observed (unregistered) activities comprises four categories: underground activities, illegal activities, activities undertaken by households for their own final use, and activities that are non-observed because of deficiencies in the data collection programme. Underground activities differ from the other unregistered activities because they consist in the production *and* sale of goods and services that are perfectly legal, but they are concealed from the public authorities, for example to avoid paying taxes. Household activities are legal, but in most cases there is no sale of goods and services, as the household is at the same time both producer and consumer.

The distinction between unregistered, underground and household sector is important

[8] Source: Zizza (2002).

[9] For the purposes of this analysis, "unregistered" or "non-observed" can be used interchangeably.

because the closest the real business cycle literature has got to modelling unregistered activities is by modelling the household sector. Models that incorporate the household sector into real business cycle theory are Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). In both these models, households have preferences over a market good, a nonmarket (or household) good, and leisure. Only the market good can be traded. The household good can only be consumed, as it cannot be used to finance the consumption of the market good, or invested into capital.

It is immediate then to see that, by removing the assumption of nontradability of the nonmarket good, the household sector in these RBC models becomes the analogue of the unregistered sector, minus the illegal activities and the statistical error, whose role can be assumed to be minor. This is the approach that is followed here. As noted beforehand, Italy has a large underground (and thus unregistered) economy, and this might contribute to explaining why the volatility of hours is high. What is more, although none of the papers mentioned above introduced a household sector to explicitly increase the volatility of hours, their results seem encouraging for this strategy.

In fact, Benhabib, Rogerson, and Wright show that in their real business cycle model with a household sector market hours have higher volatility, compared to the same model without a household sector. This happens because in addition to the standard motive (capital accumulation) for increasing market hours when market productivity is high, there is an additional incentive to substitute home production with market production, as the latter gives higher returns.

Also, Greenwood and Hercowitz find that their model with household production generates series with higher volatility of output and market time than a prototypical real business cycle model. They explain that the inclusion of household capital enhances labour's responsiveness, as a result of the fact that both household and business capital needs to be adjusted after a shock. It must be pointed out that in their model the technology shock increases the marginal productivity of both household and business capital.

On the empirical side, a closely related work on the influence of the household sector on hours is Hall (1997). According to Hall, the most important driving force of changes in working hours are labour-supply shifts. He also suggests that non-market activities, such as job-search or home production, are the possible causes for these labor-supply shifts, and that the behaviour of hours in the data cannot be justified by relying entirely on technology shifts and intertemporal (capital accumulation) mechanisms.

The model

There is one consumption good in the economy, which can be produced either in the registered sector or in the unregistered sector. As in the previous model, hiring and firing costs prevent firms from adjusting the level of employment fully in response to a shock, but now households can allocate their time between leisure, registered activities and unregistered activities. There is a continuum of measure one of agents, equally endowed with one unit of time. The utility function of each agent is:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] ,$$

where l_t is leisure and c_t is consumption.

As in the previous section, there is a fixed cost of participation in registered activities, while there is no participation cost for unregistered activities. Employed agents are those working in the registered sector. Both the unemployed and the employed participate in unregistered activities: h_{uUt} is time spent by the unemployed in unregistered activities, and h_{eUt} and h_{eRt} denote the time allocated by the employed to, respectively, unregistered and registered activities. Individuals choose employment lotteries and either there is a market where individuals can insure themselves, or employment insurance is provided by firms. The variable n_t is the probability of being employed, $1 - n_t$ is the probability of being unemployed: by the Law of Large Numbers, since there exists a continuum of agents of measure one, n_t is also the fraction of employed agents, that is, those working in registered activities.

Output in registered activities is given by:

$$y_{Rt} = z_{Rt} k_{Rt}^{\alpha} n_t^{1-\alpha} h_{eRt} ,$$

where z_{Rt} is the exogenous technology shock and k_{Rt} is capital used in registered activities. All agents have access to production in unregistered activities, which takes place according to a constant elasticity production function:

$$y_{Ut} = z_{Ut} \left\{ \eta k_{Ut}^{\xi} + (1-\eta) h_{Ut}^{\xi} \right\}^{\frac{1}{\xi}} ,$$

where z_{Ut} is the exogenous shock, specific to the unregistered sector. Capital (consumer durables) and labour used in unregistered activities are k_{Ut} and $h_{Ut} = (1 - n_t) h_{uUt} +$

$n_t h_{eUt}$. $(1-\xi)^{-1}$ gives the elasticity of substitution between capital and labour. It is assumed that all output produced in the underground sector is analogous to output produced in the registered sector. In any given period it can be consumed or stored and transformed into capital. Consumption is allocated between the employed and the unemployed according to the equation:

$$c_t = (1-n_t)\, c_{ut} + n_t c_{et} \ . \tag{3.4}$$

Total capital is the sum of capital used in the registered and unregistered activities, and it evolves accordingly to the following laws of motion:

$$k_{Rt+1} = (1-\delta_R)\, k_{Rt} + i_{Rt} \ , \tag{3.5}$$

$$k_{Ut+1} = (1-\delta_U)\, k_{Ut} + i_{Ut} \ , \tag{3.6}$$

where δ_R and δ_U are depreciation rates on the two types of capital. k_{R0} and k_{U0} are given, as well as employment at time -1, n_{-1}.

Firms pay adjustment costs whenever they lay off workers or recruit new ones:

$$AC_t = -\frac{\phi}{2}\left(n_t - n_{t-1}\right)^2 \ . \tag{3.7}$$

Output produced in the underground economy is tradeable and it is sold at the same price as output produced in registered activities. The aggregate budget constraint is

$$c_t + i_t = z_{Rt} k_{Rt}^{\alpha} n_t^{1-\alpha} h_{eRt} + z_{Ut}\left\{\eta k_{Ut}^{\xi} + (1-\eta)\, h_{Ut}^{\xi}\right\}^{\frac{1}{\xi}} - \frac{\phi}{2}\left(n_t - n_{t-1}\right)^2 \ , \tag{3.8}$$

where $i_t = i_{Rt} + i_{Ut}$. The logarithms of the exogenous shocks follow two distinct AR(1) processes:

$$\log z_{Rt+1} = \rho_R \log z_{Rt} + \varepsilon_{Rt+1} \ , \tag{3.9}$$

$$\log z_{Ut+1} = \rho_U \log z_{Ut} + \varepsilon_{Ut+1} \ . \tag{3.10}$$

As explained by Appendix 3.A, under some conditions the decentralised solution coincides with the solution to the social planner's problem, which maximises the sum of agents' utilities:

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} (1-n_t) \left[\frac{(c_{ut})^{1-\theta}-1}{1-\theta} + A\frac{(1-h_{uUt})^{1-\gamma}-1}{1-\gamma}\right] \\ +n_t \left[\frac{(c_{et})^{1-\theta}-1}{1-\theta} + A\frac{(\psi-h_{eRt}-h_{eUt})^{1-\gamma}-1}{1-\gamma}\right] \end{array} \right\},$$

subject to (3.4)-(3.10).

Optimality requires the marginal utilities for consumption and leisure of both the employed and the unemployed to be equal, therefore, since preferences are separable, employed and unemployed agents consume the same amount of c_t and enjoy the same level of leisure. The first order conditions can be written as follows:

$$(c_{et})^{-\theta} = (c_{ut})^{-\theta} = (c_t)^{-\theta}$$

$$A(1-h_{uUt})^{-\gamma} = (c_t)^{-\theta} \frac{\vartheta y_{Ut}}{\vartheta h_{Ut}}$$

$$A(\psi - h_{eRt} - h_{eUt})^{-\gamma} = (c_t)^{-\theta} z_{Rt} k_{Rt}^{\alpha} n_t^{-\alpha} = (c_t)^{-\theta} \frac{\vartheta y_{Ut}}{\vartheta h_{Ut}}$$

$$(c_t)^{-\theta} = \beta E_t \left[(c_{t+1})^{-\theta} \left(1 - \delta_R + \alpha z_{Rt} k_{Rt}^{\alpha-1} n_t^{1-\alpha} h_{eRt}\right)\right]$$

$$(c_t)^{-\theta} = \beta E_t \left[(c_{t+1})^{-\theta} \left(1 - \delta_R + \frac{\vartheta y_{Ut+1}}{\vartheta k_{U,t+1}}\right)\right]$$

$$\begin{aligned} &A\frac{(\psi - h_{eR} - h_{eU})^{1-\gamma} - 1}{1-\gamma} - A\frac{(1-h_{uU})^{1-\gamma} - 1}{1-\gamma} = \\ &= (c_t)^{-\theta} \left[(1-\alpha) z_{Rt} k_{Rt}^{\alpha} n_t^{-\alpha} h_{eRt} + \frac{\vartheta y_{Ut}}{\vartheta h_{Ut}} (h_{eUt} - h_{uUt}) - \phi(n_t - n_{t-1})\right] + \\ &+ \beta E_t \left[(c_{t+1})^{-\theta} \phi(n_{t+1} - n_t)\right] \end{aligned}$$

Table 3.4: Parameter values

α	β	$\delta_R = \delta_U$	θ	γ	n	h_{eR}	h_{eU}	$k_U \,/\, h_U$	$\rho_R = \rho_U$
0.33	0.99	0.0088	1	2	0.37	0.32	0.25	11.63	0.95

Calibration

Because of the lack of statistics on the underground economy, only a subset of the parameters may be estimated on the basis of a priori information. There is no information about the degree of substitutability between labour and capital in unregistered activities. The performance of the model and its ability to capture real world statistics can therefore vary a lot, but this does not constitute a drawback of the model, because several alternative versions can be tested against each other, according to alternative parameter specifications.

The capital share in registered activities α, the discount rate β, and the depreciation rate δ_R are set at the same level used in the previous sections. During simulation exercises, no interesting features emerged by varying the intertemporal elasticities of substitution of leisure and consumption, therefore they have been set to $\theta = 1$ and $\gamma = 2$, because these values were used in the other sections and seemed to match better the moments in the data. The depreciation rate of capital used in the underground sector is assumed equal to the one of capital employed in registered activities.

The four parameters A, a, ψ and η are obtained from the steady state equations in order to match four observations: the fraction of population that works n, the fraction of time devoted to registered activities h_{eR} and unregistered activities h_{eU} by employed households, and the capital/hours ratio in the underground economy. The values for h_{eU} and $k_U \,/\, h_U$ are taken from Greenwood, Rogerson, and Wright (1995), n and h_{eR} are set as in the previous calibration exercises. The autocorrelation coefficients ρ_R and ρ_U are set equal to 0.95, as in much of the literature. As in the models of the previous sections, the standard deviation of the exogenous shocks is calibrated so as to match the standard volatility of output. It is also assumed that the shock in the underground sector mimics the shock in the registered sector ($\rho_R = \rho_U$). The parameter values and the steady state observation used for calibrating the model are reported in Table 3.4.

The parameter ξ remains to be specified, together with the correlation between the innovations ε_{Rt} and ε_{Ut}. There is little evidence to be used as a guide in the choice of these

Table 3.5: Evaluation of the model for different parameter values

	Relative standard deviation			
	n	h_{eR}	c	i
Model 1	0.98	0.22	0.43	1.17
Model 2	0.99	0.44	0.37	0.98
Model 3	0.89	0.18	0.43	3.77
Model 4	1.02	0.30	0.18	3.46

All models use α = 0.33, β = 0.99, $\delta_R = \delta_U$ = 0.0088, θ = 1, γ = 2, ϕ = 0.2, $\rho_R = \rho_U$ = 0.95.
a, A, ψ, and η are determined so that n = 0.37, h_{eR} = 0.32, h_{eU} = 0.25, k_U / h_U = 11.63.
Model 1 sets ξ = -0.6 and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to -1.
Model 2 sets ξ = -2 and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to -1.
Model 3 sets ξ = -0.6 and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1.
Model 4 sets ξ = -2 and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1.

parameters, but several alternatives can be formulated and tested against each other, so that the behaviour of the model can be depicted under some selected scenery.

Results

Table 3.5 lists some summary statistics for several versions of the model. To allow comparison with the data, investment is investment in the two capital stocks, consumption is consumption of goods produced in the registered sector and hours worked are those in the registered sector. The magnitude of the standard deviations relative to output of the variables in the model depends crucially on two aspects: the incentive to move across sectors, as measured by the correlation between shocks, and the degree of substitutability between labour and capital in the underground economy, as measured by the parameter ξ. Table 3.5 reports the results of simulations with a common adjustment cost of 0.2, and different, "extreme" values of those parameters. Higher or lower values introduced complex roots in the approximated solution, and therefore they were discarded.

Models 1 and 2 represent the cases of high incentives to move across sectors, because the correlation between shocks is equal to minus one, and Models 3 and 4 of low incentives, because the correlation between shocks is equal to one. Model 1 and 3 describe a situation of high substitutability of capital and hours in the underground economy, whereas Models 2 and 4 a situation of low substitutability.

The effect of incentives to move across sectors on the volatility of the labour input is entirely predictable: when incentives to move are minimal there is less volatility of

Table 3.6: Standard deviations and correlations with output

	Relative standard deviation			
	n	h_{eR}	c	i
$\phi = 0.30$	0.99	0.63	0.33	3.23
$\phi = 0.31$	0.94	0.76	0.31	3.08
$\phi = 0.32$	0.87	0.94	0.27	2.78
$\phi = 0.33$	0.75	1.17	0.17	2.22

Model 2 sets ξ = -1 and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1. All the other parameters are as in Table 3.5.

labour. Table 3.5 simply states that when the incentives increase labour adjustment is made through hours when hours and capital are low substitutes, and through employment when hours and capital are high substitutes. However when the correlation between shocks is equal to minus one there is less volatility of investment. This happens because investment creates capital for future periods, but when shocks are negatively correlated and persistent there is less need to change total investment, because investment in one of the capital stocks can simply replace the other.

Different elasticities of substitution between capital and hours in the underground economy have an impact on hours, consumption and investment. When substitutability is low there must be a larger increase in hours worked after a shock in the underground economy, therefore the volatility of hours is higher. As in the models of the previous sections, when the labour supply is more rigid there is less consumption smoothing. When substitutability is high adjustment can be made through investment in unregistered activities instead of hours, and this explains why the volatility of investment is higher. In conclusion, substitutability, and incentives to move have different effects on the fluctuations of the variables of the model, and trying to reproduce the Italian data is difficult, as for example an increase in the volatility of hours can be made only at the expense of that of consumption.

The ability of the model to capture real-world statistics can be tested by making some choice for the parameters values. Since the main feature of the Italian business cycle is the high volatility of hours with respect to employment, parameters were chosen with the aim to match as closely as possible this empirical fact, avoiding at the same time generating a too low a volatility of consumption and too high a volatility of investment. Results obtained for that particular parametrization are in Table 3.6.

Results are very sensitive to the choice of ϕ, but higher values of the adjustment cost introduced complex roots in the model. The model performs better than the one in the previous section in matching the relative volatility of investment, but the increase in the volatility of the labour input is done at the expense of the volatility of consumption, which becomes too low. This phenomenon can be explained as in Section 2.5 by intertemporal substitution. After a positive technology shock, households spread the payoff of working longer hours into the future. The result is lower relative volatility of consumption, because consumption moves less than one for one with output. This intertemporal substitution effect is less pronounced when the adjustment takes place via changes in n instead of h; probably this happens because the need to insure the unemployed limits the possibility of spreading the gain in production into future periods.

3.4 Conclusion

This chapter modifies a RBC model by adding two important features of the Italian economy, namely the labour market rigidities and the underground sector. The rationale for this modification is the high volatility of hours worked in Italy, which cannot be captured by a standard RBC model, as explained in the previous chapter.

Unfortunately, Italy lacks definite information on hours of work, since the only data available is for large industrial firms. As a result, the possibility of establishing whether or not "RBC theory can explain business cycle fluctuations in Italy" is diminished: the Solow residuals could not be computed, as the data on hours are affected by measurement error. However, even though the available evidence is insufficient, it cannot be regarded as completely devoid of significance, furthermore, the relevance of the RBC theory can only increase if it is generalised to countries having "extreme" (i.e. different from the US) fluctuations. Italy is not the only country where a significant proportion of the adjustments in the labour input is made through changes in hours: according to Burdett and Wright's calculations[10], the percentage of the variation in the labour input that is due to hours (as opposed to number of employees) is considerably lower in the US and Canada than in the main European economies.

With these considerations in mind, finding the "perfect" match for the volatility of hours worked in Italy is probably less important than putting the RBC theory to a test.

[10](1989), page 1493.

The introduction of labour adjustment costs and the unregistered sector had the desired effect of augmenting the volatility of hours, and therefore it can be considered as partially successful. The reason why success is not full is the behaviour of consumption: as in the previous chapter, when the standard deviation of hours increases, the standard deviation of consumption falls, becoming much lower than in the actual data.

Of course, the institutional differences between Italy and the US are not limited to labour market rigidities and the underground sector. Therefore, there may be other modelling strategies that can bring full success and at the same time be "realistic", in the sense of giving a stylized yet accurate description of Italy's distinctive economic environment. What's more, a RBC researcher could say that per capita hours in Italy do not necessarily have a very high volatility because of "institutions" or "imperfections" in the labour market. They may fluctuate so markedly because individuals want them to, since all fluctuations are optimal responses to exogenous shocks, given the environment. Thus, the answer may lie in preferences, but then this line of attack is left to future research.

Appendix 3.A: Social planner versus competitive equilibrium solution

Throughout Chapter 3 the social planner problem is used to find the decentralised or competitive equilibrium solution of all three RBC models. To see whether this is an appropriate strategy I will look at each problem individually. In short, it is possible to prove that the social planner solution coincides with the decentralised solution under some conditions: (i) lotteries are added to the individuals' consumption sets; (ii) there is a market for contingent contracts, allowing agents to seek insurance of income risk; (iii) we make some specific assumptions on the behavior of firms.

The model with firing costs and separable utility, pages 59-64

This model is a simpler version of the model built by Kydland and Prescott, "Hours and Employment Variation in Business Cycle Theory", Economic Theory, 1991. It is simpler because: (i) I do not consider time to build new capital; (ii) I do not include inventory investment in the production function; and (iii) I do not assume that workweek lengths are different commodities, and households are constrained to choose only one workweek length. Instead, households can supply several workweek lengths to several plants at the same time.

The environment

There is a continuum of households indexed by $i \in [0,1]$. The utility function of each households is:

$$U^i = E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(c_t^i, l_t^i\right) \ .$$

The function U has the form:

$$U\left(c_t^i, l_t^i\right) = \left[\frac{\left(c_t^i\right)^{1-\theta} - 1}{1-\theta} + A\frac{\left(l_t^i\right)^{1-\gamma} - 1}{1-\gamma}\right] \ ,$$

where c_t^i is consumption and l_t^i is leisure. The time endowment of each household is normalised to 1, but if the household decides to work she loses for commuting an amount of time equal to $1-\psi$, where ψ is a fixed parameter. Consequently, the amount of leisure that the household enjoys is equal to 1 if she does not work, and if she works it is equal to $\psi - h_t^i$, where h_t^i are hours worked.

Capital is owned by households and its law of motion is:

$$k_{t+1} = (1 - \delta) k_t + i_t \ . \tag{3.11}$$

There is a continuum of firms, all symmetric and indexed by $j \in [0, 1]$. Production takes place in plants that are operated during a number of hours that is the same for all workers. The plant's production function is:

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t \ ,$$

where h denotes the workweek, n employment in the plant and k the amount of capital in the plant. The variable z is the exogenous stochastic process for technology. As stated above, all the n workers at the plant work h units of time.

Kydland and Prescott (1991) use this production function to introduce adjustment along the intensive and the extensive margin, since both the time a plant can be operated and the number of workers operating a plant can be varied. We can see that, given the workweek, plants are subject to constant returns to scale. Prescott's (2001) motivation for assuming a Cobb-Douglas in capital and employment is that the labour share of product is constant in the data, even though the price of labour relative to capital has increased dramatically.

With this production function the marginal productivity of employment is decreasing, while the marginal productivity of hours is constant. Osuna and Ríos-Rull (2003) explain this by the presence of teamwork, which introduces imperfect substitutability between employment and hours. Teamwork means that a plant can only be operated when all its workers are present. When a plant changes its workweek, the amount of capital available to each worker does not change, therefore the marginal productivity of hours is constant. On the other hand, when a plant increases the size of its labor force, the amount of capital available to each worker decreases, therefore the marginal productivity of employment is decreasing.

I assume that each firm can have several plants with different workweeks, and new plants can be opened at zero cost. However, it is straightforward to show that each firm will have one or several plants, but will choose only one workweek. In fact, given constant returns in bodies and capital, plants with n workers and a workweek of h hours can split at no cost into n units of one employee that works h hours. Moreover, consider for example plant A and plant B both belonging to the same firm, and assume with no loss of generality that both plants have only one employee and $h_A > h_B$. The firm takes the rental rate of capital as given, and allocates capital across plants so as to equalise the marginal productivity of capital in each plant to the rental rate of capital, as shown by equation (3.30). In fact, equation (3.30) shows that capital per worker is a function of the rental rate of capital and the workweek length. Thus, plants can be indexed by their workweek and their capital per worker. Since it is optimal for the firm to allocate more capital in plant A rather than B (cf. again equation 3.30), this means that the marginal product of hours will be greater in plant A rather than B. As a result, $w'(h_A) > w'(h_B)$ (equation 3.32), but since h_A and h_B are perfect substitutes from the household' point of view, only plant A will be operational. Thus, each firm may have one or several plants, but only one workweek length.

Firms pay quadratic adjustment cost whenever workers are laid off or new workers are recruited:

$$AC_t^j = \frac{\phi}{2}\left(n_t^j - n_{t-1}^j\right)^2 ,$$

where AC^j denotes the adjustment cost paid by firm j.

Since households have to use an amount of time ψ before they provide any labour service, the household choice set is not convex. Therefore, following Hansen (1985) and Rogerson (1988), I assume that households have access to employment lotteries.

Households choose a probability of working, and a lottery allocates the households between employment and unemployment. Several arrangements are possible to construct a competitive equilibrium with lotteries where households are insured from income risk. For example, Prescott and Townsend (1984) assume that insurance is provided by production firms, while Hansen (1985) assumes perfectly competitive insurance firms. I will assume here that insurance is provided by production firms, and then I will show that the first order conditions in my model are the same for the social planner and the competitive equilibrium with lotteries.

In every period t, a lottery determines which households are to be employed and which households are to be unemployed. Since the set of firms and the set of households have both measure one, and firms are all symmetric, the probability of being employed is simply denoted with n_t, which is equal to the fraction of employed households over the total number of households. Moreover, prior to the resolution of uncertainty, households trade in contracts which provide units of consumption contingent on the realization of the lottery.

Pareto optimal allocation

The Pareto optimal allocation corresponds to the solution of the following social planning problem:

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left\{ (1-n_{t+j}) \left[\frac{\left(c_{t+j}^u\right)^{1-\theta} - 1}{1-\theta} \right] + n_{t+j} \left[\frac{\left(c_{t+j}^e\right)^{1-\theta} - 1}{1-\theta} + A \frac{(\psi - h_{t+j})^{1-\gamma} - 1}{1-\gamma} \right] \right\} ,$$

subject to:

$$(1-n_t)\, c_t^u + n_t c_t^e + i_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t - \frac{\phi}{2} (n_t - n_{t-1})^2 \ ,$$

$$k_{t+1} = (1-\delta)\, k_t + i_t \ ,$$

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1} \ ,$$

where c_t^u is consumption of the unemployed and c_t^e is consumption of the employed, and ε_t is i.i.d. $N\left(0, \sigma^2\right)$. A fraction n_t of the households work, and the remaining $(1-n_t)$ do not. The social planner maximises the weighted sum of agent's utilities, subject to the resource constraint of the economy.

The first order conditions are:

$$(c_t^u)^{-\theta} = (c_t^e)^{-\theta} \ , \tag{3.12}$$

$$A\,(\psi - h_t)^{-\gamma} = c_t^{-\theta} z_t k_t^{\alpha} n_t^{-\alpha} \ , \tag{3.13}$$

$$c_t^{-\theta} = \beta E_t \left[c_{t+1}^{-\theta} \left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta \right) \right] \ , \tag{3.14}$$

$$-A \frac{(\psi - h_t)^{1-\gamma} - 1}{1-\gamma} = c_t^{-\theta} (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} h_t - c_t^{-\theta} \phi (n_t - n_{t-1}) + \beta E_t \left[c_{t+1}^{-\theta} \phi (n_{t+1} - n_t) \right] \ . \tag{3.15}$$

Competitive Equilibrium

The rental rates of labour and capital are denoted by w_t (wage per person) and r_t respectively. Each household chooses a lottery where with probability n_t she works and with probability $1 - n_t$ she does not work. Hence, with probability n_t her income is $w_t + r_t k_t$ and with probability $1 - n_t$ her income is $r_t k_t$. However, households can seek insurance by ex-ante trade in contingent contracts, which deliver units of consumption contingent on the outcome of the lottery.

Following Prescott and Townsend (page 40), let us assume that production firms supply these contingent contracts. That is, firms commit to supply to the households, at a given price, units of consumption conditional upon the outcome of the lottery. Alternatively, these contracts could be supplied by intermediaries, for example insurance firms. I will loosely refer to these contracts between households and firms as both "insurance contracts" and "contingent contracts".

The timing of the events is as follows: (i) at the start of period t the exogenous shock occurs, therefore the variable z_t is publicly observed; (ii) firms and households trade in contingent contracts; (iii) agents determine their optimal levels of y_t, c_t^u, c_t^e, h_t, i_t^u, i_t^e, $k_{e,t+1}$, $k_{u,t+1}$, and the probability of working n_t; (iv) a lottery allocates households between employment and unemployment; (v) firms deliver units of consumption to households contingent on their type. Therefore, we can see that the competitive equilibrium with lotteries rests crucially on the lottery being publicly observable and the insurance contract enforceable.

Later, we will see that in the competitive equilibrium with lotteries there is perfect insurance of income risk, so initially identical households are also identical ex-post. This result, together with the assumption that firms are symmetric, guarantees the equality between the "individually chosen" n_t and the aggregate probability n_t, because identical households make identical choices. On the other hand, the equality between the probability n_t and the fraction or number of households that work is guaranteed by the law of large numbers (Uhlig (1996)).

Let q_t denote the number of commitments to supply x_t^θ units of output to the bearer (household) who announces she is of type θ, while p_t denotes the unit price of such contingent contract. Firms only need to provide insurance to unemployed households, so it is possible to take the normalisation $x_t^e = 0$ and $x_t^u = 1$. Then, for each firm, the following condition must be satisfied:

$$z_t k_t^\alpha n_t^{1-\alpha} h_t - r_t k_t - w_t n_t - \frac{\phi}{2}\left(n_t - n_{t-1}\right)^2 + p_t q_t - q_t\left(1 - \lambda_t\right) \geq 0 \ , \tag{3.16}$$

where λ_t is the probability of a household being employed. Equation (3.16) states that each firm cannot plan to redistribute more than the profit from production. The price of current output is normalised to 1, and price of period-t consumption in terms of period-t output is also 1. Each firm takes λ_t as given. The market for contingent contracts is a spot market.

Beforehand, we denoted the probability of a household being employed with n_t, so it is tempting to write in (3.16) $\lambda_t = n_t$. However, the equivalence between the competitive equilibrium with lotteries and the Pareto-optimal allocation depends upon the requirement that firms fail to recognise (or commit not to recognise) that their choice of n_t influences the amount of insurance they offer. Firms choose the optimal number of workers by maximising profits with respect to n_t, given n_{t-1}, but they take the probability λ_t of the condition (3.16) as given.

A reasonable justification for this behaviour is that n_t is the number of workers that the individual firm employs (a choice variable), but the λ_t in (3.16) is an aggregate probability that is beyond the control of the firm. An alternative approach (for example, Hansen on page 325) assumes that the contingent contracts are offered by perfectly competitive insurance firms who take n_t as given.

Perfect competition and price-taking behaviour in the market for contingent contracts are crucial assumptions, so we must also assume that both households and firms take the price of the contingent contract as given. Let p_t be the price of the contract. Prior to the lottery, each firm acts to maximise profits from the sale of insurance contracts:

$$\pi_t = p_t q_t - q_t (1 - \lambda_t) \ ,$$

which gives:

$$p_t = 1 - \lambda_t \ ,$$

and $\pi_t = 0$. Notice that both firms and households choose the level of insurance taking λ_t as given, but the insurance contracts specify that $\lambda_t = n_t$, therefore it is also true that:

$$p_t = 1 - n_t \ , \tag{3.17}$$

(however, n_t is chosen after q_t is decided). In this environment, unemployment is the only source of heterogeneity. We assume that households are all identical at time zero. The objective of the household is to maximise ex-ante expected utility:

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left\{ (1 - \lambda_{t+j}) \left[\frac{\left(c_{t+j}^u\right)^{1-\theta} - 1}{1-\theta} \right] + \lambda_{t+j} \left[\frac{\left(c_{t+j}^e\right)^{1-\theta} - 1}{1-\theta} + A \frac{(\psi - h_{t+j})^{1-\gamma} - 1}{1-\gamma} \right] \right\} \ , \tag{3.18}$$

subject to:

$$c_t^e + i_t^e = w_t (h_t) + r_t k_t - p_t q_t \quad \text{with probability } \lambda_t \ , \tag{3.19}$$

$$c_t^u + i_t^u = r_t k_t + q_t - p_t q_t \quad \text{with probability } (1 - \lambda_t) \ , \tag{3.20}$$

$$k_{e,t+1} = (1 - \delta) k_t + i_t^e \quad \text{with probability } \lambda_t \ , \tag{3.21}$$

$$k_{u,t+1} = (1 - \delta) k_t + i_t^u \quad \text{with probability } (1 - \lambda_t) \ . \tag{3.22}$$

We want to abstract from the initial distribution of capital in the economy, therefore we assume a particular case where $k_{e,t} = k_{u,t}$. That is, households are all identical ex-ante. However, we will show that the trade in contingent contracts results in perfect insurance, therefore households are also identical ex-post and the above maximisation problem is always valid. Note that the wage per person depends on the number of hours worked, due to the nature of the plant technology.

There are 4 state variables in the above problem: individual capital k, aggregate capital (let's denote it by $\overline{k}$), the aggregate probability λ and the exogenous state z. The aggregate variables $\overline{k}$ and λ affect the prices of labour and capital, but they are beyond the control of the household. The dynamic programming problem is:

$$\begin{aligned} V\left(k,\overline{k},\lambda,z\right) &= \max_{c^u,c^e,k'_u,k'_e,i^e,i^u,q} \\ &(1-\lambda)\left\{\frac{(c^u)^{1-\theta}-1}{1-\theta}+\beta EV\left(k'_u,\overline{k}',\lambda',z'\right)\right\} \\ &+\lambda\left\{\frac{(c^e)^{1-\theta}-1}{1-\theta}+A\frac{(\psi-h)^{1-\gamma}-1}{1-\gamma}+\beta EV\left(k'_e,\overline{k}',\lambda',z'\right)\right\} , \end{aligned}$$

subject to the above constraints. The first order conditions are:

$$(c^u)^{-\theta} = \beta EV_1\left(k'_u,\overline{k}',\lambda',z'\right) , \tag{3.23}$$

$$(c^e)^{-\theta} = \beta EV_1\left(k'_e,\overline{k}',\lambda',z'\right) , \tag{3.24}$$

$$(c^u)^{-\theta} = (c^e)^{-\theta} , \tag{3.25}$$

where V_1 is the derivative of V with respect to its first argument. Equation (3.25) implies that $c^u_t = c^e_t = c_t$. This, together with eqs. (3.23) and (3.24), imply that $k'_e = k'_u$, therefore $i^e_t = i^u_t$. As a result, the left-hand sides of eqs. (3.19) and (3.20) are identical. So, q will be chosen so that the right-hand sides are equal as well: $q_t = w_t$. That is, households will choose to insure themselves fully. Because of complete insurance, ex-ante identical households are also identical ex-post, so $\overline{k}'_e = \overline{k}'_u = k_{t+1}$.

Then, the expected amount of resources available to the household is:

$$c_t + i_t = r_t k_t + w_t\left(h_t\right) n_t , \tag{3.26}$$

and the law of motion for capital is the same as (3.11). After the trade in contingent contracts has taken place, households will maximise (3.18) subject to (3.26) and (3.11), substituting λ_t with n_t. The first-order conditions (3.23) and (3.24) can be written as:

$$c_t^{-\theta} = \beta E_t \left[c_{t+1}^{-\theta} \left(1 + r_{t+1} - \delta\right) \right] , \tag{3.27}$$

the first-order condition with respect to n_t is:

$$c_t^{-\theta} w_t \left(h_t\right) = -A \frac{\left(\psi - h_t\right)^{1-\gamma} - 1}{1-\gamma} , \tag{3.28}$$

and the first-order condition with respect to h_t is:

$$n_t A \left(\psi - h_t\right)^{-\gamma} = c_t^{-\theta} w_t' \left(h_t\right) n_t . \tag{3.29}$$

From the maximisation problem of the firm we get:

$$\max_{k_t,\, n_t} E_t \sum_{j=0}^{\infty} \beta^j Q_{t,t+j} \left[z_t k_t^{\alpha} n_t^{1-\alpha} h_t - r_t k_t - w_t \left(h_t\right) n_t - \frac{\phi}{2} \left(n_t - n_{t-1}\right)^2 \right] ,$$

where $\beta^j Q_{t,t+j}$ is the relevant discount factor between t and $t+j$. Since firms are owned by households, $Q_{t,t+j} = \frac{u'(C_{t+j})}{u'(C_t)} = \frac{c_{t+j}^{-\theta}}{c_t^{-\theta}}$. The first order conditions with respect to k_t, n_t and h_t are:

$$r_t = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha} h_t , \tag{3.30}$$

$$w_t \left(h_t\right) = \left(1-\alpha\right) z_t k_t^{\alpha} n_t^{-\alpha} h_t - \phi \left(n_t - n_{t-1}\right) + \beta E_t \left[\frac{c_{t+1}^{-\theta}}{c_t^{-\theta}} \phi \left(n_{t+1} - n_t\right) \right] , \tag{3.31}$$

$$w_t' \left(h_t\right) n_t = z_t k_t^{\alpha} n_t^{1-\alpha} , \tag{3.32}$$

and thus:

$$w_t \left(h_t; r_t\right) = \left(1-\alpha\right) z_t^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{1}{1-\alpha}} - \phi \left(n_t - n_{t-1}\right) + \beta E_t \left[\frac{c_{t+1}^{-\theta}}{c_t^{-\theta}} \phi \left(n_{t+1} - n_t\right) \right] ,$$

$$w_t'(h_t; r_t) = z_t^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\alpha}{1-\alpha}} .$$

These first-order conditions and expressions for w and r are analogous to the ones presented by Alpanda and Ueberfeldt (2004) on page 22. We can note that in the steady state w and r are homogeneous of degree zero in (k, n), thus the assumption of perfect competition is consistent.

Now, it is easy to see that (3.12)-(3.15) can be obtained from (3.25)-(3.32). In conclusion, the solution of the social planner problem coincides with the solution of the competitive equilibrium with lotteries and complete insurance of income risk. Since the competitive equilibrium with lotteries and complete insurance is Pareto optimal, it then follows that there is no incentive for ex-post (after the lottery) trading between employed and unemployed households.

Some assumptions on the behaviour of the firm are crucial to ensure the equivalence between the competitive and the social planner solutions. Firms act as price-takers as their size is small with respect to the market, and since plants are ordered by workweek length and capital per worker (as explained on page 77), wages per person are a function of hours and the rental rate of capital. Because of these behavioural assumptions, we are able to obtain our equivalence result, in spite of the increasing returns to scale technology and the possible externality (due to the labour adjustment costs), which are all potentially problematic.

The model with firing costs and nonseparable utility, pages 64-65

The only difference between this model and the model on pages 59-64 is the momentary utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^{\mu} l_t^{1-\mu}\right)^{1-\theta} - 1}{1-\theta} \right] .$$

The economic environment is the same as before: the plant technology and the labour adjustment costs are the same. Firms may have one or several plants, but for the reasons explained on page 77 they will choose only one workweek length. The lottery and the market for contingent contracts is introduced in the same way as before, with the same unfolding of events. Contingent contracts are provided by production firms, which behave like perfectly competitive insurers and take the aggregate probability λ_t as given. As a result, the price of a contingent contract (which delivers one unit of the consumption good to the unemployed) is:

$$p_t = 1 - \lambda_t = 1 - n_t \ .$$

The first order conditions of the social planner's problem are:

$$(c_t^u)^{\mu-\mu\theta-1} = (c_t^e)^{\mu-\mu\theta-1} (\psi - h_t)^{(1-\mu)(1-\theta)} \ , \tag{3.33}$$

$$(c_t^u)^{\mu-\mu\theta-1} = \beta E_t \left[(c_{t+1}^u)^{\mu-\mu\theta-1} \left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta\right)\right] \ , \tag{3.34}$$

$$\begin{aligned} \frac{(c_t^u)^{\mu(1-\theta)} - 1}{1-\theta} - \frac{\left((c_t^e)^{\mu} (\psi - h_t)^{1-\mu}\right)^{1-\theta} - 1}{1-\theta} - \mu (c_t^u)^{\mu-\mu\theta} + \\ + \mu (c_t^e)^{\mu-\mu\theta} (\psi - h_t)^{(1-\mu)(1-\theta)} = \\ = \mu (c_t^u)^{\mu-\mu\theta-1} (1-\alpha) z_t k_{t-1}^{\alpha} n_t^{-\alpha} h_t - \mu (c_t^u)^{\mu-\mu\theta-1} \phi (n_t - n_{t-1}) + \\ + \beta E_t \left[\mu \left(c_{t+1}^u\right)^{\mu-\mu\theta-1} \phi (n_{t+1} - n_t)\right] \ , \end{aligned} \tag{3.35}$$

$$z_t k_{t-1}^{\alpha} n_t^{-\alpha} = \frac{1-\mu}{\mu} \frac{c_t^e}{\psi - h_t} \ . \tag{3.36}$$

Households are initially identical, and before the lottery takes place they choose their level of insurance so as to maximise ex-ante expected utility:

$$\begin{aligned} V\left(k, \overline{k}, \lambda, z\right) \;=\; & \max_{c^u, c^e, k_u', k_e', i^e, i^u, q} \\ & (1-\lambda) \left\{ \frac{[(c^u)^{\mu}]^{1-\theta} - 1}{1-\theta} + \beta EV\left(k_u', \overline{k}', \lambda', z'\right)\right\} \\ & + \lambda \left\{ \frac{\left[(c^e)^{\mu} (\psi - h)^{1-\mu}\right]^{1-\theta} - 1}{1-\theta} + \beta EV\left(k_e', \overline{k}', \lambda', z'\right)\right\} \ , \end{aligned}$$

subject to the constraints (3.19)-(3.22), where the expression for p has been substituted out. The first-order conditions are:

$$\mu\left(c^{u}\right)^{\mu-\mu\theta-1}=\beta EV_{1}\left(k_{u}^{\prime},\overline{k}^{\prime},\lambda^{\prime},z^{\prime}\right)\ , \tag{3.37}$$

$$\mu\left(c^{e}\right)^{\mu-\mu\theta-1}(\psi-h)^{(1-\mu)(1-\theta)}=\beta EV_{1}\left(k_{e}^{\prime},\overline{k}^{\prime},\lambda^{\prime},z^{\prime}\right)\ , \tag{3.38}$$

$$\left(c^{u}\right)^{\mu-\mu\theta-1}=\left(c^{e}\right)^{\mu-\mu\theta-1}(\psi-h)^{(1-\mu)(1-\theta)}\ . \tag{3.39}$$

Equations (3.37) to (3.39) together imply that $k_{u}^{\prime}=k_{e}^{\prime}$. Therefore, from the budget constraints of the employed and unemployed households, it is possible to find out an expression for q:

$$q=w\left(h\right)-c^{e}+c^{u}\ .$$

The proof that $k_{u}^{\prime}=k_{e}^{\prime}$ is presented at the end of this Section. This implies that $i_{u}=i_{e}$, since households have the same amount of capital initially. Then, it is possible to see that, with nonseparable utility, there is no perfect insurance of income risk, as the consumption of the unemployed is different from the consumption of the employed. However, initially identical households will have the same level of capital in each period.

The expected amount of resources available is the same for all households. After substituting the exogenous λ_{t} with n_{t}, it is given by:

$$\left(1-n_{t}\right)c_{t}^{u}+n_{t}c_{t}^{e}+i_{t}=r_{t}k_{t}+w_{t}\left(h_{t}\right)n_{t}\ .$$

The first-order conditions of the optimal control problem, once the level of insurance has been substituted in, are:

$$\left(1-n_{t}\right)\left(c_{t}^{u}\right)^{\mu-\mu\theta-1}=\beta E_{t}\left[\left(1-n_{t+1}\right)\left(c_{t+1}^{u}\right)^{\mu-\mu\theta-1}\left(1+r_{t+1}-\delta\right)\right]\ , \tag{3.40}$$

$$-\left\{\frac{\left[(c_t^u)^{\mu}\right]^{1-\theta}-1}{1-\theta}\right\}+\left\{\frac{\left[(c_t^e)^{\mu}(\psi-h_t)^{1-\mu}\right]^{1-\theta}-1}{1-\theta}\right\}+$$

$$+\mu\left(c_t^u\right)^{\mu-\mu\theta-1}\left(w_t\left(h_t\right)-c_t^e+c_t^u\right)=0\ , \tag{3.41}$$

$$w_t'\left(h_t\right)=\frac{1-\mu}{\mu}\frac{c_t^e}{\psi-h_t}\ . \tag{3.42}$$

As the production side is unchanged, the first-order conditions of the firm are the same as before, (3.30)-(3.32). But (3.30)-(3.32) together with (3.39)-(3.42) imply (3.33)-(3.36), therefore, the equivalence result between the competitive equilibrium with lotteries and the social planner solution is valid also in the case of nonseparable preferences. However, in this case there is no complete insurance, since $c_t^e \neq c_t^u$.

Proof that $k_u' = k_e'$: The form of the value function V is unknown, but the derivative with respect to its first argument can be recovered by the envelope theorem:

$$\begin{aligned} V_1\left(k,\overline{k},\lambda,z\right) &= \mu\left(1+r-\delta\right)\left(c^u\right)^{\mu-\mu\theta-1}\ , \\ &= \mu\left(1+r-\delta\right)\left(c^e\right)^{\mu-\mu\theta-1}\left(\psi-h\right)^{(1-\mu)(1-\theta)}\ . \end{aligned}$$

For example, substituting in the first expression c^u and leading it forward we obtain:

$$V_1\left(k',\overline{k}',\lambda',z'\right)=\mu\left(1+r'-\delta\right)\left(-k_u''+(1-\delta)\,k'+rk'+q'\lambda'\right)^{\mu-\mu\theta-1}\ , \tag{3.43}$$

where k_u'' is a function of $\left(k',\overline{k}',\lambda',z'\right)$. Since the first-order conditions of the household maximisation problem require that $V_1\left(k_u',\overline{k}',\lambda',z'\right)=V_1\left(k_e',\overline{k}',\lambda',z'\right)$, it is clear that this is possible only if $k_u'=k_e'$. The same would be true if we used the second expression for V_1.

The model with an underground sector and firing costs, pages 68-73

Now both employed and unemployed households have access to production in unregistered activities, which have a CES production function:

$$y_{Ut} = z_{Ut}\left\{\eta k_{Ut}^{\xi} + (1-\eta)\, h_{Ut}^{\xi}\right\}^{\frac{1}{\xi}} \ ,$$

where z_{Ut} is the exogenous shock, specific to the unregistered sector. The variables k_{Ut} and h_{Ut} are total capital and labour used in unregistered activities, and they are supplied by both employed and unemployed households:

$$k_{Ut} = \int_0^{\lambda} k_{eUt}\, di + \int_{\lambda}^{1} k_{uUt}\, di \ ,$$

$$h_{Ut} = \int_0^{\lambda} h_{eUt}\, di + \int_{\lambda}^{1} h_{uUt}\, di \ .$$

where k_{eU} (h_{eU}) is capital (labour) supplied to the unregistered sector by the employed, and k_{uU} (h_{uU}) are supplied by the unemployed households. Employed households are those working in the registered sector. For convenience, I assume that profits or losses in unregistered activities are distributed equally among all households, and that each productive factor is paid according to its (aggregate) marginal productivity:

$$\pi_{Ut} = y_{Ut} - \frac{\vartheta y_{Ut}}{\vartheta k_{Ut}} k_{Ut} - \frac{\vartheta y_{Ut}}{\vartheta h_{Ut}} \left[\lambda_t h_{eUt} + (1-\lambda_t)\, h_{uUt}\right] \ .$$

The CES production function may not be realistic, but little is known about the underground economy, and the CES functional form allows me more flexibility in the simulations.

The form of momentary utility function and the assumptions on the production firms are the same as in the model of pages 59-64. Therefore, the first-order conditions of the firms' optimization problem, and the price of the contingent contract are the same as before. Production firms are all registered.

Households are assumed to be initially identical, so $k_{uU} = k_{eU} = k_U$ and $k_{uR} = k_{eR} = k_R$. Households take the marginal productivities $\frac{\vartheta y_U}{\vartheta k_U}$ and $\frac{\vartheta y_U}{\vartheta h_U}$ as given. They choose the level of insurance so as to solve the following problem:

$$V\left(k_R, k_U, \mathbf{x}\right) = \max$$
$$= (1-\lambda)\left\{\frac{\left(c^u\right)^{1-\theta}-1}{1-\theta} + A\frac{\left(1-h_{uU}\right)^{1-\gamma}-1}{1-\gamma} + \beta EV\left(k'_{uR}, k'_{uU}, \mathbf{x}'\right)\right\} +$$
$$+\lambda\left\{\frac{\left(c^e\right)^{1-\theta}-1}{1-\theta} + A\frac{\left(\psi - h_{eR} - h_{eU}\right)^{1-\gamma}-1}{1-\gamma} + \beta EV\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right)\right\} ,$$

subject to:

$$c^e + k'_{eR} + k'_{eU} = \left(1 + r - \delta_R\right) k_R + \left(1 + \frac{\vartheta y_U}{\vartheta k_U} - \delta_U\right) k_U + w\left(h_{eR}\right) + \frac{\vartheta y_U}{\vartheta h_U} h_{eU} + \pi_U - (1-\lambda) q \quad \text{with p. } \lambda$$

$$c^u + k'_{uR} + k'_{uU} = \left(1 + r - \delta_R\right) k_R + \left(1 + \frac{\vartheta y_U}{\vartheta k_U} - \delta_U\right) k_U + \frac{\vartheta y_U}{\vartheta h_U} h_{uU} + \pi_U + q\lambda \quad \text{with prob. } (1-\lambda) ,$$

where $\mathbf{x} = \left[\overline{k}_R, \overline{k}_U, \lambda, z\right]$ is a vector of aggregate state variables, and the δ's are depreciation rates.

The first-order conditions with respect to the capital stocks next period and q are analogous to the ones found beforehand:

$$\left(c^u\right)^{-\theta} = \beta EV_1\left(k'_{uR}, k'_{uU}, \mathbf{x}'\right) = \beta EV_2\left(k'_{uR}, k'_{uU}, \mathbf{x}'\right) , \tag{3.44}$$

$$\left(c^e\right)^{-\theta} = \beta EV_1\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right) = \beta EV_2\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right) , \tag{3.45}$$

$$\left(c^u\right)^{-\theta} = \left(c^e\right)^{-\theta} . \tag{3.46}$$

The last equation implies that $c^e = c^u = c$ in every period. By the envelope theorem:

$$V_2\left(k'_{uR}, k'_{uU}, \mathbf{x}\right) = \left(c'\right)^{-\theta}\left[1 + \frac{\vartheta y'_U}{\vartheta k'_U} - \delta_U\right] = V_2\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right) ,$$

$$V_1\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right) = \left(c'\right)^{-\theta}\left[1 + r' - \delta_R\right] = V_1\left(k'_{eR}, k'_{eU}, \mathbf{x}'\right) ,$$

where $\frac{\vartheta y'_U}{\vartheta k'_U} = (z'_U)^{\xi}\, \eta \left(\frac{k'_U}{y'_U}\right)^{\xi-1}$. The above two equations show that households in each period equalise the expected returns from investing in both types of technology, the registered and the unregistered. Since r' is given for both employed and unemployed households, and $\frac{\vartheta y'_U}{\vartheta k'_U}$ does not depend on the labour choices of the individual household, then it must be true that $i_{uU} = i_{eU}$, but this this implies that $k'_{uU} = k'_{eU}$. Therefore, $k'_{uR} = k'_{eR}$. As a result, q will be chosen in the following way:

$$q = w\left(h_{eR}\right) + \frac{\vartheta y_U}{\vartheta h_U}\left(h_{eU} - h_{uU}\right) \ ,$$

therefore, the expected amount of resources available is the same for all households and it is equal to:

$$\begin{aligned} c_t + k_{R,t+1} + k_{U,t+1} &= \left(1 + r_t - \delta_R\right) k_{Rt} + \left(1 + \frac{\vartheta y_{Ut}}{\vartheta k_{Ut}} - \delta_U\right) k_{Ut} + \\ &+ n_t\left(w\left(h_{eRt}\right) + \frac{\vartheta y_{Ut}}{\vartheta h_{Ut}} h_{eUt}\right) + \left(1 - n_t\right)\frac{\vartheta y_{Ut}}{\vartheta h_{Ut}} h_{uUt} + \pi_{Ut} \ . \end{aligned}$$

By analogy with the other models, we can now conclude that also in this model with an underground sector the decentralised and the social planner solution are equivalent, provided we introduce lotteries and a market for contingent contracts, and provided we assume that k_U and h_U are paid their (aggregate) marginal productivity.

Chapter 4

The Relative or Allocative Effects of Shocks in Open Economies

4.1 Introduction

This chapter investigates how exogenous monetary, productivity and government expenditure shocks affect relative quantities and relative prices in an open economy. It does so by means of a two-country general equilibrium model, with monopolistic competition and sticky prices. Therefore, the model presented in this chapter is akin to the "new open economy macroeconomics" literature initiated by Obstfeld and Rogoff's (1995) *Redux* model, and recently surveyed by Lane (2001) and Sarno (2001). The relative effects under investigation are those between tradeable and nontradeable goods within the same country.

So far, the theoretical new open economy literature has not focused on the consequences of exogenous shocks for relative prices and quantities within one country. However, even if we restrict the attention to monetary shocks only, there is some empirical evidence that clearly indicates that money has heterogeneous effects on the industries or sectors of the economy. For example, and without any claim to provide an exhaustive list, the following papers look at the sectoral effects of either systematic monetary policy or monetary policy shocks: Barth and Ramey (2001), Cimadomo (2003), Dedola and Lippi (2000), Ganley and Salmon (1997), Peersman and Smets (2005), and Raddatz and Rigobon (2003). All these papers find that the responses to monetary policy vary across sectors. But the empirical study that is most closely related to this chapter is Llaudes (2003), who isolates the differential effects that monetary policy and exchange rate fluctuations have on industry and the service sector of the economy[1].

Some of these empirical papers on the industry or sectoral effects of monetary policy also look at the potential explanations. For example, Cimadomo finds a positive and strong correlation between the sensitivity to systematic monetary policy and the degree of price rigidity in a sector, while Dedola and Lippi find that the responses to monetary policy shocks are related to some industry characteristics such as output durability, financing requirements and borrowing capacity. None of this papers looks at the degree of openness or tradability at the sectoral level. Necessarily, applied research focuses on explanations that have already been put forward by theory. For example, a key assumption for money to have real effects is the presence of nominal rigidities, so if they vary across sectors, then monetary policy has heterogenous effects. On the other hand, Dedola and Lippi's suggested explanation is founded on the theories that emphasize the importance of the cost-channel effect of monetary policy and of financial frictions.

By investigating the consequences of exogenous, and therefore also money, shocks for relative prices and quantities, this chapter suggests another possible explanation of the heterogeneous effects of monetary policy shocks, namely the degree of tradability at the sectoral level. In fact, in new open economy models, monetary shocks affect the relative price of tradeables over nontradeables, and the relative quantity of tradeables over nontradeables. Intuitively, this can happen both because nominal exchange rate effects alter the relative prices within the country, and because international wealth transfers induce the consumers to reallocate their expenditures.

However, this chapter does not focus on monetary shocks only, since other shocks also cause relative or allocative effects. The argument is that, with the introduction of nontradeable goods, new open economy models have become able to generate predictions about the movement of relative prices and quantities, which can be compared with the actual data. Therefore, by looking at relative prices and output it is possible to better understand the models, or test them along other dimensions. To increase understanding,

[1] Llaudes finds that the response of manufacturing (tradeable) to a monetary policy shock is significantly different to the response of the service (nontradeable) sector. However, from the quantitative point of view his results cannot be directly compared to the ones presented in Section 4.5. In his paper, shocks are identified as innovations to domestic short-term interest rates, and shocks to the exchange rate are considered as different from money shocks (small open economy assumption). Moreover, the tradable sector is likely to contain nontradable components and vice-versa, and both sectors most likely use foreign nontradeables as production inputs.

this chapter uses general functional forms where possible, and analyses how sensitive are the results to the choice of parameter values.

In contrast to previous new open economy models with tradeables and nontradeables[2], the model of this chapter has the following novel features. First, in order to analyse the impact of shocks on the sectoral allocation of employment, it is assumed that individuals cannot simultaneously work in both sectors. However, they can adjust both hours of work and participation rates. Second, the model considers a full range of exogenous shocks: nominal (money), supply (productivity) and demand (government expenditure). Both productivity and government expenditure are sector-specific, but the effects of aggregate shocks of the same nature can be analysed very easily. Third, in order to obtain a relative supply curve with an upward slope, this chapter explores the possibility that the marginal labour productivity is decreasing.

The main results are as follows. Shocks have small effects on the relative price of domestic tradeables versus nontradeables, and on the ratios of domestic tradeable versus nontradeable output and employment. These effects change in both sign and magnitude under alternative parametrizations. Monetary shocks may affect relative prices and relative output. Labour productivity is crucial in understanding how tradeable-nontradeable prices and output are affected by shocks.

The rest of the chapter is organised in the following way. Section 4.2 presents the model, and Section 4.3 describes the solution method. The calibration of the model is described in Section 4.4. Section 4.5 reports and explains the findings. Finally, Section 4.6 concludes with some additional remarks.

4.2 The model

The world economy consists of two countries of equal size, named Home and Foreign, that engage in the production and trade of differentiated goods for final consumption. Consumers purchase a variety of goods and for each differentiated good there exists a downward-sloping demand curve. Goods can be tradeable or nontradeable. In the Home country, a continuum of unit size of tradeables and a continuum of unit size of nontradeables are produced, the same is true for the Foreign country. Therefore, each country has two sectors, one for the production of tradeables and one for nontradeables.

[2]The first new open economy model with tradeables and nontradeables is due to Hau (2000).

The model possesses two standard features of the new open-economy macroeconomics framework: nominal rigidities and monopolistic competition. While the assumption of nominal rigidities is at the root of the short run real effects of money, the assumption of monopolistic competition simply introduces a wedge between prices and marginal costs. As we shall see, in the short-run output is demand-determined, but prices do not adjust fully, so there is the possibility that firms may prefer to stop production because the marginal cost becomes higher than the price. The assumption of monopolistic competition allows us to neglect this possibility, at least for "sufficiently small" shocks[3].

As in Hau (2000) and Obstfeld and Rogoff (2000), there is imperfect competition in both the goods and labour markets. Each individual is a monopolistic supplier of her own differentiated labour, and firms need a variety of labour inputs in production. The assumption of monopolistic competition in both markets allows us to contemplate the contemporaneous introduction of two sorts of nominal rigidities, price and wage rigidities. This has been done, for example, by Chari, Kehoe and MacGrattan (2002); others like Corsetti and Pesenti (2001), Obstfeld and Rogoff (2000) and Hau (2000) introduce only wage rigidity. However, because the analysis of the interactions between price and wage rigidities goes beyond the purpose of the chapter, and in order not to complicate the model too much, the model presented here has only price rigidity. This modelling strategy follows the *Redux* and it is probably still the most popular one in the new open economy literature.

A crucial assumption in the model is that individuals cannot contemporaneously supply their labour to the production of both tradeable and nontradeable goods. Instead, they can work only in one sector at a time. The standard assumption in new open economy models with tradeables and nontradeables is that the two sectors pay the same wage for the same labour type, and individuals can work contemporaneously in both sectors. This assumption is made, for example, in Benigno and Thoenissen (2003), Cavelaars (2001), Hau (2000), Obstfeld and Rogoff (2000). In this way, the maximization problem is the same for all individuals, and there is no heterogeneity due to sectoral allocations and wage differentials. However, we shall see that this sort of heterogeneity is easy to work with, provided preferences are separable and there are complete domestic markets. Moreover, this sort of heterogeneity has a potential advantage, because it allows the model to be extended to include imperfections in the allocation of individuals across sectors, for

[3] Obstfeld and Rogoff (1996) and Corsetti and Pesenti (2001) discuss this issue in detail.

example due to costly sectoral mobility or search frictions. These could be introduced in a rather straightforward manner. However, the model presented in this chapter does not introduce moving costs.

The model also considers the possibility of decreasing marginal labour productivity. The standard assumption of the new open economy literature is constant marginal labour productivity and constant returns to scale, but in general, because of the assumption of imperfect competition, it is possible to depart from this assumption. As compared to the case of constant marginal labour productivity, decreasing marginal labour productivity has different implications for the relative supply curve (the relationship between the ratio of tradeable to nontradeable output and the relative price of tradeables to nontradeables).

As in Chari, Kehoe and McGrattan (2002), Home and Foreign money supplies are exogenous random variables. The other sources of uncertainty in the economy are government expenditure and productivity shocks. The model abstracts from long run growth, which implies that only temporary shocks are considered.

Firms and the labour market

Due to the assumption of monopolistic competition and the restriction that individuals can only work in one sector, there is some tedious notation to be introduced beforehand. The aim is nevertheless to increase clarity.

In each country and in each sector a continuum of firms exist, each of them producing a single differentiated product. That is, in the Home country there is a continuum of unit size of firms in sector TH and a continuum of unit size of firms in sector N. The firms and the goods they produce are indexed by $f_{TH} \in [0,1]$ for the Home tradeable sector and $f_N \in [0,1]$ for the Home nontradeable sector. In the Foreign country, they are indexed by $f_{TF}^* \in [0,1]$ and $f_N^* \in [0,1]$. Each country is populated by a continuum of unit size of individuals. Individuals in the Home country are indexed by $i \in [0,1]$, and individuals in the Foreign country are indexed by $i^* \in [0,1]$. All foreign variables, sets and indexes are indicated by stars.

Firms need a variety of labour inputs in production, but each individual supplies labour to one sector only. Let us start with the Home country, and denote by $I_{TH,t}$ and $I_{N,t}$ the sets of individuals at date t supplying, respectively, the tradeable and non tradeable goods sectors. That is:

$$I_{TH,t} = \{i \in [0,1] : h_{TH,t}(i) > 0\} \ ,$$

$$I_{N,t} = \{i \in [0,1] : h_{N,t}(i) > 0\} \ .$$

Then, the restriction that individuals can only work in one sector is equivalent to the requirement $I_{TH,t} \cap I_{N,t} = \emptyset$, but $(I_{TH,t} \cup I_{N,t}) \subset [0,1]$, since some individuals are not working. The individuals' allocative choices will determine the "size" or probability measure of sectors TH and N. Let us call these measures $m(I_{TH,t})$ and $m(I_{N,t})$, and assume that the production functions for each individual firm in sectors TH and N at date t are respectively:

$$y_{TH,t}(f_{TH}) = z_{TH,t} \cdot \widetilde{h}_{TH,t}(f_{TH})^{\alpha} \ , \tag{4.1}$$

$$y_{N,t}(f_N) = z_{N,t} \cdot \widetilde{h}_{N,t}(f_N)^{\alpha} \ . \tag{4.2}$$

The variable $\widetilde{h}_{TH}(f_{TH})$ $(\widetilde{h}_N(f_N))$ is an aggregate of all labour inputs used by firm f_{TH} (f_N) in sector TH (N), z_{TH} and z_N represent technology, and α is a parameter which allows for decreasing marginal labour productivity. The variables z_{TH} and z_N affect labour productivity and the logs follow exogenous stochastic processes.

The aggregators $\widetilde{h}_{TH}(f_{TH})$ and $\widetilde{h}_N(f_N)$ arise because of imperfect competition in the labour market. Labour inputs are imperfect substitutes in production, with constant elasticity of substitution η_1. Given the allocative choices of individuals, firms at date t combine individual labour inputs in the following way:

$$\widetilde{h}_{TH,t}(f_{TH}) = \left[\left(\frac{1}{m(I_{TH,t})}\right)^{\frac{1}{\eta_1}} \int_{i \in I_{TH,t}} h_{TH,t}(i, f_{TH})^{\frac{\eta_1 - 1}{\eta_1}} \, di\right]^{\frac{\eta_1}{\eta_1 - 1}} ,$$

$$\widetilde{h}_{N,t}(f_N) = \left[\left(\frac{1}{m(I_{N,t})}\right)^{\frac{1}{\eta_1}} \int_{i \in I_{N,t}} h_{N,t}(i, f_N)^{\frac{\eta_1 - 1}{\eta_1}} \, di\right]^{\frac{\eta_1}{\eta_1 - 1}} .$$

The corresponding wage indexes at date t for each sector are:

$$W_{TH,t} = \left(\frac{1}{m(I_{TH,t})} \int_{i \in I_{TH,t}} w_{TH,t}(i)^{1-\eta_1} \, di\right)^{\frac{1}{1-\eta_1}} ,$$

$$W_{N,t}=\left(\frac{1}{m\left(I_{N,t}\right)}\int_{i\in I_{N,t}}w_{N,t}\left(i\right)^{1-\eta_1}di\right)^{\frac{1}{1-\eta_1}}.$$

Wages for differentiated labour inputs are the same for firms within the same sector (that is, $w_{TH}\left(i,f_{TH}\right)=w_{TH}\left(i\right)$). Firms take the individuals' allocative choice and supply of hours as given.

Cost minimization implies that the demands of firms f_{TH} and f_N for each differentiated labour input are:

$$h_{TH,t}\left(i,f_{TH}\right)=\left(\frac{1}{m\left(I_{TH,t}\right)}\right)\left(\frac{w_{TH,t}\left(i\right)}{W_{TH,t}}\right)^{-\eta_1}\widetilde{h}_{TH,t}\left(f_{TH}\right)\quad i\in I_{TH}\ , \tag{4.3}$$

$$h_{N,t}\left(i,f_{N}\right)=\left(\frac{1}{m\left(I_{N,t}\right)}\right)\left(\frac{w_{N,t}\left(i\right)}{W_{N,t}}\right)^{-\eta_1}\widetilde{h}_{N,t}\left(f_{N}\right)\quad i\in I_{N}\ . \tag{4.4}$$

There are parallel production functions, labour input aggregators and labour demands for Foreign-produced tradeables and for Foreign nontradeables, with the same elasticity of substitution η_1 and parameter α.

Individual preferences and budget constraints

There is no possibility of migration across countries, but individuals can move costlessly from one sector to the other within each country. Individuals derive utility from consumption, real money balances, government expenditure and leisure. Utility is separable. Real money balances are included in the utility function because of the liquidity services that they provide. Individuals can smooth their consumption by holding three assets: a domestic state-contingent bond, an international non-state-contingent real bond, and money, but they do not invest in physical capital.

The assumption of no investment in physical capital is very common in new open economy models[4], therefore it is also made here. The inclusion of capital may or may not alter the transmission of shocks in these models, at least along some dimensions. For example, Chari, Kehoe and McGrattan (2002) found that almost all of the movements in output come from variations in labour, with little or no impact from physical investment. More generally, models without capital are unsuitable for analyzing the impact of money

[4]See, for example, Obstfeld and Rogoff (1995).

shocks on investment fluctuations at business cycle frequencies, and for answering certain types of policy questions.

This section starts with a description of the preferences, then moves to the individual budget constraints, and finally concludes with the first-order conditions from the individual maximization problem.

Preferences

As shown in Section 4.2, individual labour supplies have distinctive characteristics and are only imperfect substitutes in production. This allows each individual to act as a monopolistic supplier of her own labour effort, taking into account the labour demands from firms in her maximization problem.

Each individual is endowed with one unit of time, a fraction of which can be supplied as labour either to the tradeable goods sector or to the nontradeable goods sector. Moreover, as in Burnside, Eichenbaum and Rebelo (1993), any individual who works incurs a fixed participation cost, measured in units of foregone leisure. The standard assumption in new open economy models with tradeables and nontradeables is that individuals can work contemporaneously in both sectors. In this model labour mobility is restricted, because labour services cannot be contemporaneously supplied to both the tradeable and nontradeable goods sector. The advantage of having this restriction is that the model can be nested in a broad class of models with imperfections in the allocation of individuals across sectors, with causes ranging from costly sectoral mobility to search frictions. This is not possible in the models where individuals can work contemporaneously in both sectors[5]. However, since sectors do not need to pay the same wage, this restriction introduces heterogeneity in the model.

Nonetheless, this type of heterogeneity can be easily dealt with by applying Rogerson's (1988) result for sectoral economies. It basically states that, under the assumptions of separable utility function and complete domestic markets[6], if individuals can choose the probabilities of working in sectors, then the decentralized equilibrium reproduces the

[5]Like, for example, Benigno and Thoenissen (2003), Cavelaars (2001), Hau (2000), Obstfeld and Rogoff (2000). These models assume that sectors pay the same wage. Wage differentials may arise, for example, because of costly sectoral mobility or search frictions, or because the disutility of working in the two sectors is different.

[6]Rogerson (1988) does not explicitly introduce complete markets but this assumption is implicit in his analysis. In fact, he introduces lotteries to convexify the individual's consumption sets, and he assumes that some market arrangements allow the individuals to perfectly share income risks. It is immediate to interpret these arrangements as trade in state-contingent claims.

socially optimal allocation. As a result, if these assumptions hold, it is not necessary to keep track of the heterogeneity among individuals while solving for this decentralized equilibrium. This happens because optimal allocations for initially identical individuals imply that the marginal utility of consumption is equal for all individuals; since utility is separable this implies that consumption levels must be equal for all individuals. Of course, in this decentralized equilibrium the wage that the individual receives depends on the sector she ends up in, but the assumption of complete markets is sufficient to ensure that individuals are all ex-ante identical in each period.

Following Rogerson, the probabilities of working in each sector are added to the individual maximization problem. That is, the utility of a Home individual of type i is written as follows:

$$U(i) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} u\left(C_t(i)\right) + L\left(\frac{M_t(i)}{P_t}\right) + F\left(G_{TH,t}, G_{N,t}\right) + n_{TH,t}(i)\, v\left(\Gamma - \psi - \mathbf{h}_{TH,t}(i)\right) \\ + n_{N,t}(i)\, v\left(\Gamma - \psi - \mathbf{h}_{N,t}(i)\right) + \left(1 - n_{TH,t}(i) - n_{N,t}(i)\right) v(\Gamma) \end{array} \right] \tag{4.5}$$

where C is an aggregate consumption index, $\frac{M}{P}$ are real money balances, G_{TH} and G_N are Home government aggregate spending in tradeables and nontradeables, n_{TH},n_N are the probabilities of working in the tradeable and nontradeable sectors, Γ is the total time available, ψ is a fixed cost of participation, the same for all individuals, and $\mathbf{h}_{TH}$ and $\mathbf{h}_N$ are the total hours supplied to sectors TH and N, that is:

$$\mathbf{h}_{TH}(i) = \int_0^1 h_{TH}(i, f_{TH})\, df_{TH} \ , \tag{4.6}$$

$$\mathbf{h}_N(i) = \int_0^1 h_N(i, f_N)\, df_N \ . \tag{4.7}$$

The functions u, L, F and v are assumed to be three times continuously differentiable, concave and strictly increasing. Individuals take government expenditures as given[7].

Separable preferences of this form have been used, among others, by Corsetti and Pesenti (2001) and Obstfeld and Rogoff (1995, 2000). If it is coupled with the assumption of Cobb-Douglas preferences over tradeable goods, this form of momentary utility

[7]Government expenditures are included in the utility function to allow a direct effect on individual welfare. The case of wasteful government expenditure is a special case, where $F(\cdot) = 0$.

function has the advantage of removing permanent wealth effects after a shock, because international borrowing and lending does not affect equilibrium allocations[8].

Foreign preferences are similarly written:

$$U^*(i^*) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} u\left(C_t^*(i^*)\right) + L\left(\frac{M_t^*(i^*)}{P_t^*}\right) + F\left(G_{TF,t}^*, G_{N,t}^*\right) \\ +n_{TF,t}^*(i^*)\, v\left(\Gamma - \psi - \mathbf{h}_{TF,t}^*(i^*)\right) \\ +n_{N,t}^*(i^*)\, v\left(\Gamma - \psi - \mathbf{h}_{N,t}^*(i^*)\right) + \left(1 - n_{TF,t}^*(i^*) - n_{N,t}^*(i^*)\right) v(\Gamma) \end{array} \right], \tag{4.8}$$

thus assuming that both countries have the same functional forms u, L, F, v and parameters Γ, ψ.

At this point, there is one more aspect of this setup that is worth highlighting. Rogerson's device (i.e. the addition of lotteries to the consumption set) and the assumption of complete domestic markets produce the convenient result that it is not necessary to keep track of the heterogeneity among individuals. However, it is crucial to have individuals that are always identical ex-ante, before they are allocated to sectors. If individuals are always identical ex-ante, they always make the same choices. Specifically, they all choose the same probabilities[9]. Equality ex-ante is achieved here also because wages are flexible. To understand this point, consider a model with Calvo-style wage rigidity and different wages in sectors. In this model individuals are different ex-ante because they cannot change their wage at the same time; in fact, there would be a distribution of wages in the two sectors. In this case, it is not clear whether individuals will all choose the same probabilities of working in sectors. These considerations lead to the conclusion that the addition of wage stickiness is problematic in this setup[10].

[8] However, a drawback of this setup with perfect insurance of income risk is that the unemployed enjoy greater utility ex-post than the employed, because consumption levels are the same but the unemployed have more time for leisure. It is possible to avoid this awkward implication by the adoption of a different specification of preferences, but this would entail a departure from the functional forms most commonly adopted in the literature. One of the aims of the paper is to try to make assumptions on functional forms that are quite general and already adopted in the literature. In this way it is hoped that the transmission of shocks will be more transparent, at least for those readers who are familiar with new open economy models.

[9] Notwithstanding the assumption of monopolistic competition, in reality this setup is close to a representative agent model. This is because in the end we concentrate on equilibria where individuals make the same choices.

[10] The interaction of wage stickiness with price stickiness may generate some interesting dynamics that is not just the "sum" of the two, but the advantage of focusing on only one type of nominal rigidity is that the transmission mechanism is certainly more transparent.

Individual budget constraints

At the international level, markets are incomplete: individuals trade in a real one-period non-contingent bond B, denominated in units of the Home tradeable goods consumption index. Interest is decided at the beginning of the period and paid at the end.

The budget constraint of a Home individual of type i in period t is written as follows:

$$\begin{aligned} B_t(i)\,P_{T,t} + M_t(i) \;\leq\; & (1+r_{t-1})\,B_{t-1}(i)\,P_{T,t} + M_{t-1}(i) + TR_t(i) - P_tC_t(i) + \\ & + n_{TH,t}(i)\,w_{TH,t}(i)\,\mathbf{h}_{TH,t}(i) + n_{N,t}(i)\,w_{N,t}(i)\,\mathbf{h}_{N,t}(i) + \\ & + \int_0^1 \Pi_{TH,t}(i,f_{TH})\,df_{TH} + \int_0^1 \Pi_{N,t}(i,f_N)\,df_N\ , \end{aligned} \tag{4.9}$$

where B is the internationally traded bond, M are nominal money balances, r is the real interest rate, TR are government transfers, w_{TH} and w_N are wages paid in the tradeable and nontradeable sectors, $\Pi_{TH}(i, f_{TH})$ and $\Pi_N(i, f_N)$ are the profits that the individual i receives from firms f_{TH} in the tradeable sector and f_N in the nontradeable sector. Everyone owns an equal share of all domestic firms, therefore everyone receives the same share of profits, which arise because of the imperfect competition assumption.

Note that, following Rogerson, the probabilities of working in each sector are incorporated in the budget constraint of the Home individual i. This is another way of showing that there are complete domestic markets, and as a result perfect insurance against individual income uncertainty. Because individuals are choosing the probabilities of working in sectors with different wages, their labour income will be a random variable. However, the individual income uncertainty can be eliminated by pooling it among all individuals. Because of the concavity of the utility function, this arrangement is optimal and is achieved through complete domestic markets. This is shown in the budget constraint, which states that the individual receives the expected income, given her choice of probabilities[11].

A Foreign individual of type i in period t faces the following budget constraint:

[11] As it will be explained later, in the end we concentrate on symmetric equilibria where individuals make the same choices. Then, because of the law of large numbers, the probability chosen at the individual level and the fraction of individuals at the aggregate level that work in a given sector coincide. This individual budget constraint is justified both by the assumption of complete domestic markets and by the fact that the individual takes into account the impact on her expected income when choosing the probabilities.

$$
\begin{aligned}
B_t^*(i^*)\frac{P_{T,t}}{e_t}+M_t^*(i^*) \quad \leq \quad & (1+r_{t-1})\,B_{t-1}^*(i^*)\frac{P_{T,t}}{e_t}+M_{t-1}^*(i^*)+TR_t^*(i^*)-P_t^*C_t^*(i^*)+\\
& +n_{TF,t}^*(i^*)\,w_{TF,t}^*(i^*)\,\mathbf{h}_{TF,t}^*(i^*)+n_{N,t}^*(i^*)\,w_{N,t}^*(i^*)\,\mathbf{h}_{N,t}^*(i^*)+\\
& +\int_0^1 \Pi_{TF,t}^*(i^*,f_{TF}^*)\,df_{TF}^*+\int_0^1 \Pi_{N,t}^*(i^*,f_N^*)\,df_N^*\ , \qquad (4.10)
\end{aligned}
$$

whose explanation is similar to the one given for the Home country. Naturally, an equilibrium condition in the bonds market at each date t is added to the model:

$$
\int_0^1 B_t(i)\,di+\int_0^1 B_t^*(i^*)\,di^*=0\ ,
$$

and two Fisher parity conditions define the Home and Foreign nominal interest rates:

$$
i_t\equiv E_t\left[\frac{P_{T,t+1}}{P_{T,t}}(1+r_t)\right]-1\ , \qquad (4.11)
$$

$$
i_t^*\equiv E_t\left[\frac{P_{T,t+1}}{P_{T,t}}\frac{e_t}{e_{t+1}}(1+r_t)\right]-1\ . \qquad (4.12)
$$

First-order conditions

Individuals in the Home country maximize (4.5) subject to (4.9), (4.3) and (4.4) and the no Ponzi games condition. They aggregate hours as in (4.6) and (4.7). The first-order conditions that describe the optimal allocations are as follows:

Euler equation for consumption:

$$
u'(C_t(i))\frac{P_{T,t}}{P_t}=\beta E_t\left[(1+r_t)\,u'(C_{t+1}(i))\frac{P_{T,t+1}}{P_{t+1}}\right]\ . \qquad (4.13)
$$

First-order condition with respect to money holdings:

$$
L'\left(\frac{M_t(i)}{P_t}\right)=u'(C_t(i))-\beta E_t\left[u'(C_{t+1}(i))\frac{P_t}{P_{t+1}}\right]\ . \qquad (4.14)
$$

First-order conditions with respect to hours:

$$
v'(\Gamma-\psi-\mathbf{h}_{TH,t}(i))=\frac{u'(C_t(i))}{P_t}w_{TH,t}(i)\,\frac{\eta_1-1}{\eta_1}\ , \qquad (4.15)
$$

$$v'(\Gamma - \psi - \mathbf{h}_{N,t}(i)) = \frac{u'(C_t(i))}{P_t} w_{N,t}(i) \frac{\eta_1 - 1}{\eta_1} . \tag{4.16}$$

First-order conditions with respect to the probabilities of working in sectors:

$$v(\Gamma - \psi - \mathbf{h}_{TH,t}(i)) - v(\Gamma) = -\frac{u'(C_t(i))}{P_t} w_{TH,t}(i) \mathbf{h}_{TH,t}(i) , \tag{4.17}$$

$$v(\Gamma - \psi - \mathbf{h}_{N,t}(i)) - v(\Gamma) = -\frac{u'(C_t(i))}{P_t} w_{N,t}(i) \mathbf{h}_{N,t}(i) . \tag{4.18}$$

When maximizing with respect to probabilities, individuals do not take into account the impact that their allocative choices have on the labour demands from firms, or in other words, they are "small" with respect to the market.

Analogous first-order conditions hold for the Foreign country.

Government budget constraint and money supply

The Home and Foreign governments purchase only tradeable and nontradeable goods produced in their own country. Public expenditures are financed by seigniorage revenues, and what is left is redistributed to individuals in the form of net transfers. The budget constraint of the Home government at date t is given by:

$$M_t - M_{t-1} = P_{TH,t} G_{TH,t} + P_{N,t} G_{N,t} + \int_0^1 TR_t(i)\, di . \tag{4.19}$$

G_{TH} and G_N are composite measures that aggregate the expenditures on each individual good, in the same manner as private consumption:

$$G_{TH,t} = \left[\int_0^1 g_{TH,t}(f_{TH})^{\frac{\eta_2 - 1}{\eta_2}} df_{TH} \right]^{\frac{\eta_2}{\eta_2 - 1}} ,$$

$$G_{N,t} = \left[\int_0^1 g_{N,t}(f_N)^{\frac{\eta_2 - 1}{\eta_2}} df_N \right]^{\frac{\eta_2}{\eta_2 - 1}} .$$

The variables G_{TH} and G_N can also be interpreted as public expenditure production functions. The Foreign government budget constraint and public expenditure aggregators are entirely analogous.

The two countries equate their own money supply to their own money demand, for example in the Home country at each date t the following equality holds:

$$M_t = \int_0^1 M_t(i)\, di \ ,$$

and an analogous equality holds in the Foreign country. The money growth rate is defined as:

$$\widehat{\mu}_t \equiv \frac{M_t - M_{t-1}}{M_{t-1}} \ .$$

As in Chari, Kehoe and McGrattan (2002), money growth rates and public expenditures in both countries are exogenous stochastic processes.

Consumption and price indexes

Individuals consume a positive amount of the nontradeable goods produced in their own country, and a positive amount of all the tradeable goods produced in both countries. Goods are imperfect substitutes in consumption, and goods are aggregated in each country into three distinct bundles, which combine the types of goods that consumers regard as "immediate alternatives". That is, consumers first allocate their expenditure between tradeables and nontradeables, then they allocate their expenditure on tradeables between Home and Foreign goods, and then they allocate their expenditures on tradeables and nontradeables among all the individual goods.

To be specific, at date t any individual i in the Home country has preferences over tradeable and nontradeable goods, described by the following CES aggregator:

$$C_t(i) = \left[(1-\gamma)^{\frac{1}{\phi}} \left(C_{T,t}(i) \right)^{\frac{\phi-1}{\phi}} + \gamma^{\frac{1}{\phi}} \left(C_{N,t}(i) \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \ ,$$

where $(1-\gamma)$ and γ are weights, and ϕ is the substitution elasticity. In the Foreign country, the preferences of the individual i^* are described by an equivalent aggregator, with the same parameters γ and ϕ. Note that Cobb-Douglas preferences can be obtained as a special case by setting ϕ equal to 1.

Consumers regard type-TH goods (tradeable goods produced in the Home country) as forming a separate bundle from type-TF goods (tradeable goods produced in the Foreign country), but it is assumed that type-TH and type-TF goods are "immediate alternatives" in consumption. That is, the aggregators for tradeable goods consumption in the Home and Foreign countries at date t are, respectively:

$$C_{T,t}(i) = \left[(1-\delta)^{\frac{1}{\theta}}\left(C_{TH,t}(i)\right)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}\left(C_{TF,t}(i)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

$$C^*_{T,t}(i^*) = \left[(1-\delta^*)^{\frac{1}{\theta}}\left(C^*_{TH,t}(i^*)\right)^{\frac{\theta-1}{\theta}} + (\delta^*)^{\frac{1}{\theta}}\left(C^*_{TF,t}(i^*)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

The elasticity of substitution θ between type-TH and type-TF goods is the same in both countries. The weights of Foreign-produced tradeables, δ and δ^*, can differ, implying that there can be home bias in the model. Cobb-Douglas preferences can be obtained as a special case by setting θ equal to 1.

The three consumption subindexes for the Home individual i at date t are:

$$C_{TH,t}(i) = \left[\int_0^1 c_{TH,t}(i, f_{TH})^{\frac{\eta_2-1}{\eta_2}}\, df_{TH}\right]^{\frac{\eta_2}{\eta_2-1}},$$

$$C_{TF,t}(i) = \left[\int_0^1 c_{TF,t}(i, f^*_{TF})^{\frac{\eta_2-1}{\eta_2}}\, df^*_{TF}\right]^{\frac{\eta_2}{\eta_2-1}},$$

$$C_{N,t}(i) = \left[\int_0^1 c_{N,t}(i, f_N)^{\frac{\eta_2-1}{\eta_2}}\, df_N\right]^{\frac{\eta_2}{\eta_2-1}}.$$

Foreign preferences are identical and have the same elasticity of substitution η_2. Hence, the degree of market power in the goods market can be different from the degree of market power in the labour market: η_2 can be different from η_1, but these two parameters are the same in both countries.

The Home price indexes for the three consumption baskets at date t are:

$$P_{TH,t} = \left(\int_0^1 p_{TH,t}(f_{TH})^{1-\eta_2}\, df_{TH}\right)^{\frac{1}{1-\eta_2}}, \tag{4.20}$$

$$P_{TF,t} = \left(\int_0^1 p_{TF,t}(f^*_{TF})^{1-\eta_2}\, df^*_{TF}\right)^{\frac{1}{1-\eta_2}}, \tag{4.21}$$

$$P_{N,t} = \left(\int_0^1 p_{N,t}(f_N)^{1-\eta_2}\, df_N\right)^{\frac{1}{1-\eta_2}}, \tag{4.22}$$

all in Home currency. The Foreign price indexes in Foreign currency are:

$$P^*_{TH,t} = \left(\int_0^1 p^*_{TH,t} (f_{TH})^{1-\eta_2} \, df_{TH} \right)^{\frac{1}{1-\eta_2}} , \tag{4.23}$$

$$P^*_{TF,t} = \left(\int_0^1 p^*_{TF,t} (f^*_{TF})^{1-\eta_2} \, df^*_{TF} \right)^{\frac{1}{1-\eta_2}} , \tag{4.24}$$

$$P^*_{N,t} = \left(\int_0^1 p^*_{N,t} (f^*_{N})^{1-\eta_2} \, df^*_{N} \right)^{\frac{1}{1-\eta_2}} . \tag{4.25}$$

Prices are indicated by stars when they are in Foreign currency, unstarred prices are in Home currency. The model assumes that the law of one price holds, that is, there is no international price discrimination[12]. Denoting by e_t the exchange rate (price of Home country currency in terms of the Foreign country currency) at date t, we can write:

$$p_{TH,t} (f_{TH}) = e_t \cdot p^*_{TH,t} (f_{TH}) , \tag{4.26}$$

$$p_{TF,t} (f^*_{TF}) = e_t \cdot p^*_{TF,t} (f^*_{TF}) . \tag{4.27}$$

As a result:

$$P_{TH,t} = e_t \cdot P^*_{TH,t} , \tag{4.28}$$

$$P_{TF,t} = e_t \cdot P^*_{TF,t} . \tag{4.29}$$

A Home individual i allocates optimally her expenditure within each category of goods by solving an expenditure minimization problem. Her demand functions at date t for the individual goods are:

$$c_{TH,t} (i, f_{TH}) = \left(\frac{p_{TH,t} (f_{TH})}{P_{TH,t}} \right)^{-\eta_2} C_{TH,t} (i) , \tag{4.30}$$

[12] The first paper in the new open economy literature with traded and nontraded sectors, Hau (2000), assumed instead that traded goods firms charge different prices in the Home and Foreign markets. However, in his model the law of one price is recovered, even though firms segment markets, since product prices are given by a mark-up over a fixed nominal wage.

$$c_{TF,t}\left(i, f_{TF}^{*}\right) = \left(\frac{p_{TF,t}\left(f_{TF}^{*}\right)}{P_{TF,t}}\right)^{-\eta_2} C_{TF,t}\left(i\right) \ , \tag{4.31}$$

$$c_{N,t}\left(i, f_N\right) = \left(\frac{p_{N,t}\left(f_N\right)}{P_{N,t}}\right)^{-\eta_2} C_{N,t}\left(i\right) \ , \tag{4.32}$$

with parallel demands from a Foreign individual i^*. Individual i's demand functions for the aggregate consumption bundles are:

$$C_{TH,t}\left(i\right) = \left(1-\delta\right)\left(\frac{P_{TH,t}}{P_{T,t}}\right)^{-\theta} C_{T,t}\left(i\right) \ , \tag{4.33}$$

$$C_{TF,t}\left(i\right) = \delta\left(\frac{P_{TF,t}}{P_{T,t}}\right)^{-\theta} C_{T,t}\left(i\right) \ , \tag{4.34}$$

$$C_{N,t}\left(i\right) = \gamma\left(\frac{P_{N,t}}{P_t}\right)^{-\phi} C_t\left(i\right) \ , \tag{4.35}$$

$$C_{T,t}\left(i\right) = \left(1-\gamma\right)\left(\frac{P_{T,t}}{P_t}\right)^{-\phi} C_t\left(i\right) \ . \tag{4.36}$$

$P_{T,t}$ is the Home tradeables price index, P_t is the Home consumer price index. They are given by:

$$P_{T,t} = \left[\left(1-\delta\right) P_{TH,t}^{1-\theta} + \delta P_{TF,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \ , \tag{4.37}$$

$$P_t = \left[\left(1-\gamma\right) P_{T,t}^{1-\phi} + \gamma P_{N,t}^{1-\phi}\right]^{\frac{1}{1-\phi}} \ . \tag{4.38}$$

In the Foreign country, the demands $C_{TH,t}^{*}\left(i^*\right)$, $C_{TF,t}^{*}\left(i^*\right)$, $C_{N,t}^{*}\left(i^*\right)$, $C_{T,t}^{*}\left(i^*\right)$, and the price indexes $P_{T,t}^{*}$, P_t^{*}, have the same functional forms, but with the parameter δ^* in place of δ. Since Home and Foreign consumers have different tastes for domestically versus foreign-produced traded goods, this implies, unless preferences are identical (unless $\delta = \delta^*$), that $P_{T,t} \neq e_t \cdot P_{T,t}^{*}$. Therefore, deviations from purchasing power parity arise both from differences in tastes ($\delta \neq \delta^*$) and from the presence of nontradeable goods in the consumption index.

To conclude, the Home terms of trade is defined as the ratio of the price of imports over the price of exports, in the same currency:

$$T_t \equiv \frac{P_{TF,t}}{P_{TH,t}} = \frac{P^*_{TF,t}}{P^*_{TH,t}} \,. \tag{4.39}$$

Using this definition, a fall is an "improvement" of the terms of trade, since imports become cheaper. The Foreign terms of trade is simply $T^*_t = (T_t)^{-1}$. Therefore, it is sufficient to consider only one terms of trade as the other can be immediately calculated. To ease the exposition, from now on "terms of trade" will simply refer to the Home terms of trade, i.e. the ratio $\frac{e_t P^*_{TF,t}}{P_{TH,t}}$.

Product price determination

The model introduces nominal rigidities by assuming that firms have a less than one probability of changing their prices, that is, in each period a Home firm can change its price with a fixed probability $1-\varphi$, and a Foreign firm can change its price with a fixed probability $1-\varphi^*$. As a result, the average duration of prices is fixed, and it is equal to $\frac{1}{1-\varphi}$ in the Home country and $\frac{1}{1-\varphi^*}$ in the Foreign country. This price-setting behaviour was introduced by Calvo (1983), and has the advantage of allowing the model-builder to increase or decrease the degree of nominal rigidity by simply decreasing or increasing the parameters φ and φ^*, but on the other hand, there is no theoretical justification for this behaviour, nor it is modelled endogenously[13].

Consider, for example, firm f_{TH} in the Home tradeable goods sector. Let $p_{TH,t}(f_{TH})$ denote the individual good price decided at date t (which may or may not apply at date $t+j$), let $y_{TH,t+j|t}(f_{TH})$ denote the total demand[14] for good f_{TH}, and let $\widetilde{h}_{TH,t+j|t}(f_{TH})$ denote the demand from firm f_{TH} for the aggregate labour input at date $t+j$ if the price $p_{TH,t}(f_{TH})$ decided at t still applies. At each date t, only a fraction $1-\varphi$ of firms is allowed to change its price. If the draw is "favourable" at date t and therefore firm f_{TH} is allowed to change its price, $p_{TH,t}(f_{TH})$ is chosen so as to maximize expected future discounted profits, taking into account that the probability that $p_{TH,t}(f_{TH})$ still applies at the future date $t+j$ is φ^j. Therefore, firm f_{TH} solves the following problem:

[13]Alternatively, nominal rigidities can be introduced in the model by assuming convex price adjustment costs, as in Rotemberg (1982), or by assuming convex costs of adjusting both prices and output, as in the "P-bar model" of McCallum and Nelson (1999). Inasmuch as the adjustment costs do not arise endogenously but are exogenous features, these approaches are similar in spirit to the Calvo setup. However, the dynamics may be different.

[14]This is found by integrating and summing the demand functions of the individuals and the government.

$$\begin{array}{ll} \max & E_t \sum_{j=0}^{\infty} (\varphi\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH}) + \\ -\frac{W_{TH,t+j}}{P_{t+j}} \cdot \widetilde{h}_{TH,t+j|t}(f_{TH}) \end{array} \right] \\ \text{s.t.} & y_{TH,t+j|t}(f_{TH}) = \left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}}\right)^{-\eta_2} \left(C_{TH,t+j} + C^*_{TH,t+j} + G_{TH,t+j}\right) \end{array} \quad (4.40)$$

where $Q_{t,t+j} = \frac{u'(C_{t+j})}{u'(C_t)}$, $C_{TH,t+j} = \int_0^1 C_{TH,t+j}(i)\, di$ and $C^*_{TH,t+j} = \int_0^1 C^*_{TH,t+j}(i^*)\, di^*$. Firms are owned by individuals. $\beta^j Q_{t,t+j}$ is the relevant discount factor between t and $t+j$, and it is the same for all individuals in the Home country because of the assumption of complete domestic markets. $\widetilde{h}_{TH,t+j|t}(f_{TH})$ and $y_{TH,t+j|t}(f_{TH})$ are linked by equation (4.1), and the constraint can be used to substitute $y_{TH,t+j|t}(f_{TH})$ out of the objective function. The first-order condition is as follows:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{1}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH})(1-\eta_2) + \\ +\eta_2 \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y_{TH,t+j|t}(f_{TH})} \cdot \frac{y_{TH,t+j|t}(f_{TH})}{p_{TH,t}(f_{TH})} \end{array} \right] = 0 \ , \quad (4.41)$$

where $\widetilde{h}_{TH,t+j|t}(f_{TH})$ denotes firm f_{TH}'s demand for the aggregate labour input at date $t+j$ if $p_{TH,t}(f_{TH})$ still applies. Moreover, because of the assumptions on the functional forms and the requirement $\alpha \leq 1$, profits are a concave function, therefore the first-order condition is both necessary and sufficient for an interior maximum.

The individual good demand function $y_{TH,t+j|t}(f_{TH})$ shows the absence of market segmentation from the model: firm f_{TH} sets the same price for the Home and Foreign markets. Moreover, prices are sticky in producer country currency, that is, the model assumes producer currency pricing instead of local currency pricing[15]. The absence of market segmentation (for tradeable goods) is an important assumption since the model allows for decreasing marginal labour productivity. If firm f_{TH} set two different prices, one for the Home and one for the Foreign market, the presence of decreasing marginal labour productivity would complicate the analysis because the two pricing decisions would

[15]Models that assume local currency pricing, or sticky prices in the buyer's currency, are, among many, Betts and Devereux (1996, 1998) and Chari, Kehoe and McGrattan (2003). The implications of this assumption are very serious: with full local currency pricing the degree of exchange-rate pass-through to import prices is 0, and countries are insulated from foreign monetary shocks. However, the attractiveness of this approach is that it makes possible to replicate some stylized facts about business cycle fluctuations, for example, the high variability of real and nominal exchange rates and the comovements in international consumption levels. However, Obstfeld and Rogoff (2000) present empirical evidence against full local currency pricing.

not be independent[16].

The maximization problems of Home firms f_N and Foreign firms f_{TF}^* and f_N^* are entirely analogous, the main difference is that for firms producing nontradeable goods there is no demand coming from abroad. For example, the maximization problem of firm f_N^* in the Foreign nontradeable goods sector is as follows:

$$\begin{array}{ll} \max & E_t \sum_{j=0}^{\infty} (\varphi^* \beta)^j Q_{t,t+j}^* \left[\begin{array}{c} \frac{p_{N,t}^*(f_N^*)}{P_{t+j}^*} \cdot y_{N,t+j|t}^*(f_N^*) + \\ -\frac{W_{N,t+j}}{P_{t+j}^*} \cdot \widetilde{h}_{N,t+j|t}^*(f_N^*) \end{array} \right] \\ \text{s.t.} & y_{N,t+j|t}^*(f_N^*) = \left(\frac{p_{N,t}^*(f_N^*)}{P_{N,t+j}^*} \right)^{-\eta_2} \left(C_{N,t+j}^* + G_{N,t+j}^* \right) \; , \end{array} \tag{4.42}$$

and the first-order condition is:

$$E_t \sum_{j=0}^{\infty} (\varphi^* \beta)^j Q_{t,t+j}^* \left[\frac{1}{P_{t+j}^*} \cdot y_{N,t+j|t}^*(f_N^*)(1-\eta_2) + \eta_2 \frac{W_{N,t+j}}{P_{t+j}^*} \cdot \frac{\vartheta \widetilde{h}_{N,t+j|t}^*(f_N^*)}{\vartheta y_{N,t+j|t}^*(f_N^*)} \cdot \frac{y_{N,t+j|t}^*(f_N^*)}{p_{N,t}^*(f_N^*)} \right] = 0 \tag{4.43}$$

Each Home and Foreign firm is the only supplier of its own differentiated product. The resulting market power enables the firm to earn positive profits. Indeed, in the steady state of the economy prices are a markup over the nominal marginal cost of production[17 18]. However, after a shock occurs, there are a fraction of firms φ in the Home tradeable goods sector, and a fraction φ in the Home nontradeable goods sector, that cannot adjust their prices (φ^* in the Foreign country). These firms must find optimal to supply all the output that is demanded at the price last set, that is, they must not incur losses. It has now

[16] This problem, and Obstfeld and Rogoff's remarks, motivate the decision to consider only producer currency pricing. Some authors take an "intermediate stance", and adopt a pass-through function, proposed by Corsetti and Pesenti (2001), which allows the impact of nominal exchange rates changes on consumers prices to vary.

[17] In this model firms take into account the demand for their product when maximizing profits, but they take the individuals' allocative choice and supply of hours as given.

[18] For example, if we denote by $mc(f_{TH})$ the nominal marginal cost of production of firm f_{TH}, in the steady state the following is true:
$p_{TH}(f_{TH}) = \frac{\eta_2}{\eta_2 - 1} \cdot mc(f_{TH})$
η_2 is greater than 1 by assumption. The lower is η_2, the higher are the degree of market power and the markup.
We can think of the steady state as the deterministic stationary equilibrium that is approached in the limit, as $t \to \infty$, when there is no money growth (and therefore no inflation) and all the exogenous processes are constant and equal to their expected level. The steady state coincides with the flexible price equilibrium. In fact, as $t \to \infty$, everybody has been given the chance to adjust the price. If there is no uncertainty, then at each date all firms that adjust the price set the same price.

become standard to say that this condition will always be satisfied if shocks are sufficiently "small" or the markup is sufficiently high.

Galí and Gertler (1999) have introduced an empirically-motivated alteration of the original Calvo pricing mechanism. Their assumption is that at each point in time two types of firm coexist: one type that sets prices in a forward looking, Calvo-style, manner, and the other type that sets prices in a backward looking manner. By doing so, they are able to increase the persistence of inflation. It may be interesting to ascertain whether the introduction of an additional type of nominal rigidity of this kind fundamentally alters the transmission of shocks along other dimensions, but this issue goes beyond the purpose of this chapter. The analysis is therefore focused on the original Calvo setup.

The model assumes imperfect competition with flexible wages in the labour market. However, as shown by Chari, Kehoe and McGrattan (2002), it is possible to increase the persistence (moderately) by adding sticky wages to the model[19]. Moreover, Christiano, Eichenbaum and Evans (1997) have found that firm profits seem to decrease after a monetary contraction, and they have pointed out that a model with sticky wages correctly predicts procyclical firm profits, while a model with sticky prices does not. On the other hand, the literature to date suggests that one or the other form of nominal rigidity does not alter the basic transmission of shocks to key macroeconomic variables: the impact on output and inflation, for example, is almost the same in a model with sticky prices as in a model with sticky wages[20]. Of course, it might be interesting to analyze the interaction between sticky prices and sticky wages in the transmission mechanism, but this goes beyond the purpose of the chapter. In conclusion, this chapter shares the view of Obstfeld and Rogoff (2000) that neither extreme set of assumptions is right, and that it is probably more "realistic" to adopt a combination of sticky product prices and sticky wages, but, for the purpose of the analysis of the transmission mechanism, it is better to avoid too much complexity and thus choose one assumption over the other. For the theoretical reasons explained in Section 4.2, and because price stickiness is widely adopted in the literature, this chapter introduces sticky prices as the only source of nominal rigidity, but warns the reader of their unpleasant consequences for the firms profits[21].

[19]This approach has been followed, for example, by Benigno and Thoenissen (2003).

[20]For example, this was implicit in the analysis of Corsetti and Pesenti (2001).

[21]Lane (2001) claims that the new open economy literature seems to largely emphasize price stickiness over wage stickiness, for the reasons discussed by Kimball (1995).

4.3 The solution of the model

The rest of the chapter focuses on an equilibrium where all individuals are symmetric and make the same choices, and all firms that can modify their price at date t set the same price. After having aggregated the equations describing the economy, the indexes i, i^*, f_{TH}, f_N, f^*_{TF}, f^*_N can be dropped, and variables can be reinterpreted in per-capita terms. The aggregation at the individual level is straightforward because individuals supply the same hours to the sectors, and therefore wages are the same for everyone[22]. Moreover, because of the law of large numbers, there is a correspondence between probabilities chosen at the individual level and the fraction of individuals at the aggregate level. On the other hand, firms are not all symmetric, because they set prices at different dates, and as a consequence labour demand differs from firm to firm. For those firms that cannot adjust their price at date t demand determines both output and the required labour input. However, with the Calvo pricing mechanism it is not necessary to keep track of the distribution of firms, since the aggregate equations that describe the firms pricing behaviour can be written in terms of inflation rates instead of price levels. This will be apparent in Appendix 4.A, which explicitly shows the derivation of the log-linearised equations for inflation.

In particular, an equilibrium for this economy is a collection of allocations for Home and Foreign individuals, allocations and prices for Home and Foreign firms, aggregate prices, wages, a nominal exchange rate and real and nominal interest rates that satisfy the following conditions, given the exogenous processes, initial allocations and initial prices: (i) the individual allocations solve the maximization problems of Home and Foreign individuals; (ii) Home and Foreign firms that are free to adjust prices maximize their profits; (iii) the Home and Foreign government budget constraints are satisfied; (iv) demand equals supply in all markets. At each date t, the equilibrium satisfies equations (4.1), (4.2), (4.3), (4.4) and their Foreign counterparts, equations (4.9), (4.10), (4.11) and (4.12), equations (4.13), (4.14), (4.15), (4.16), (4.17), (4.18), (4.19) and their Foreign counterparts, equations (4.20) to (4.29), equations (4.30) to (4.38) and their Foreign counterparts, equations (4.40), (4.41), (4.42), (4.43) and their counterparts, plus a no Ponzi games condition aimed at preventing unlimited borrowing.

The model cannot be solved in closed form, and a numerical approximated solution

[22] More precisely, and taking sector TH as an example, a symmetric equilibrium implies $h_{TH,t}(i, f_{TH}) = h_{TH,t}(i', f_{TH})$ at any date t. Equation (4.3) then implies $w_{TH}(i) = w_{TH}(i')$. However, $h_{TH,t}(i, f_{TH}) \neq h_{TH,t}(i, f'_{TH})$ because firms are not symmetric.

must be found instead. This is obtained by aggregating and log-linearising the abovementioned equations around a deterministic equilibrium or steady state in which all the exogenous stochastic processes are set equal to their unconditional means, their variances are set to zero, the individuals hold no internationally traded bond, and all government expenditures are equal to zero. Moreover, it is assumed that all firms within a sector have the same price[23], therefore this deterministic equilibrium coincides with a flexible price equilibrium. Moreover, in the deterministic equilibrium there is no growth in the money stock, therefore inflation is equal to zero.

As in Obstfeld and Rogoff's *Redux*, the equilibrium dynamics implied by the model is not stationary: starting from a deterministic equilibrium in which international bond holdings are equal to zero, a wide range of transitory shocks causes individuals to borrow or lend as they try to smooth their consumption over time. The wealth allocation between the two countries changes, and as a result shocks have permanent effects on the endogenous variables, which depend on this wealth allocation. This behaviour reveals itself with the presence of unit roots in the system of difference equations implied by the log-linearised model, which is not returning to the initial steady state after a shock. Therefore, the main shortcoming of nonstationarity is that it impairs the accuracy of the numerical approximated solution. In the very short run, however, this approximation certainly cannot be much worse than in an equivalent stationary model, since wealth transfers take some time to build up[24]. However, if the following three conditions are all satisfied: (i) the elasticity of substitution between tradeable and nontradeable goods is equal to the intertemporal substitution elasticity; (ii) the elasticity of substitution between Home and Foreign tradeables is equal to one; (iii) the weights of Home and Foreign goods in the tradeables consumption indexes are equal; then there are no permanent wealth effects after a shock, and the model becomes stationary. Therefore, stationarity can be embedded as

[23] With the Calvo pricing mechanism, this situation would arise at infinity because as $t \rightarrow \infty$ every firm is given the chance to adjust the price. If there is no inflation, and technology shocks are always equal to their unconditional expectation, then each period all firms that adjust the price set the same price. Therefore, if there is no uncertainty, as $t \rightarrow \infty$ the Calvo equilibrium approaches the flexible price equilibrium.

[24] Nonstationarity could be eliminated by introducing complete markets in the model, or by resorting to some other ad-hoc stationarity-inducing features, which, however, may or may not be theoretically-motivated as accurate descriptions of the economy. In the context of a small open economy, Schmitt-Grohe and Uribe (2003) show that models with or without permanent wealth effects deliver identical dynamics at business-cycle frequencies. In a two country setup, Chari, Kehoe and MacGrattan (2003) show that having complete or incomplete markets does not matter quantitatively for the volatility and persistence of real exchange rates.

a special case. The possibility of eliminating permanent wealth effects by assuming a unitary elasticity was discovered by Corsetti and Pesenti (2001).

There are ten exogenous stochastic processes in the model: two describe the behaviour of the Home and Foreign nominal money growth rates, four describe the behaviour of productivity in the Home and Foreign productions of tradeables and nontradeables, and four describe the behaviour of the Home and Foreign government expenditures in tradeables and nontradeables. All shocks to these exogenous processes are temporary. The unconditional mean of the money and government expenditure processes is zero.

Because the model has nominal money growth and inflation, nominal variables in the dynamic system of equations (the approximated solution) are divided by the appropriate price levels in order to use Uhlig's "Toolkit" algorithm (1999). The approximated solution method clearly relies on the knowledge of the steady state of the model. A close inspection of the steady state equations reveals that steady state prices can be pinned down only if the stocks of Home and Foreign money are known, and that money stocks do not affect real variables. As a result, steady state prices can be normalised at any level without affecting real variables. However, the steady state terms of trade (the price of imports over the price of exports in the same currency) and all the relative prices are not nominally indeterminate, since they depend not only on the preference parameters but also on real factors, such as the unconditional means of the productivity processes.

The determination of the steady state terms of trade follows Obstfeld and Rogoff (1995). After substituting out the variables, the optimal allocation of effort between sectors in the Home country becomes a function of Home tradeable output and the terms of trade. Analogously, the optimal allocation of effort in the Foreign country can be written as a function of Foreign tradeable output and the terms of trade. After some manipulation of the output demand equations, it is possible to find the demand for Home tradeable output relative to Foreign tradeable output (in this steady state with no international bond holdings and government expenditures) as a function of the terms of trade only. Therefore, finding the steady state terms of trade boils down to solving a system of three equations in three unknowns, given all the parameters of the model and the unconditional means of the exogenous processes. Then it is possible to compute all the coefficients of the log-linearised model, which is shown in Appendix 4.B.

4.4 Calibration

In order to make the implications of the model as transparent as possible, and closely connected to most of the new open economy literature, the functional forms adopted for preferences are quite standard. In particular, u and L are standard CRRA functions, with risk aversion parameters σ and ε respectively. Utility from leisure is given by $v(x) = \kappa \frac{x^{\omega}}{\omega}$.

The choice of parameter values allows the researcher to adapt the model to a particular setup (Euro area versus the US, for example). Parameters may be obtained for this purpose from the existing literature, or they may be estimated so as to match selected business-cycle or long-run moments in the data. However, the former strategy relies on the existence of confirmed empirical estimates, and the latter one entails some degree of freedom about the selection of moments to devote to the estimates of the parameter instead of model testing. Because of this degree of freedom, it is important to have some knowledge about how parameters affect the impact of shocks, which is one of the objectives of this chapter.

However, in practice this degree of freedom is very small for a subset of the parameters of the model, either because there is a firm consensus and the choice of moments to match is restricted by the literature, or because some parameters only affect the steady state equations but not the dynamic log-linearised equations, and therefore they have no impact on business cycle statistics and on the transmission of shocks in the model.

Therefore, it is necessary to choose beforehand the calibration of a minimal set of parameters, thus restricting the question of how shocks are transmitted only to parameters that can be chosen (in most cases) with some degree of freedom. These minimal choices are made as follows. The discount factor β is set so as to yield an annual real interest rate of four percent in the deterministic equilibrium. ψ is calibrated in the following way. In the deterministic equilibrium around which the approximation is taken, hours worked in the two Home sectors have to satisfy equations (4.15) to (4.18). Let us denote the variables in this deterministic equilibrium with a subscript 0. By combining equation (4.15) with (4.17), and (4.16) with (4.18), and after replacing the functional form for v, it is possible to obtain:

$$\frac{\kappa}{\omega}\left(\Gamma - \psi - \mathbf{h}_{TH,0}\right)^{\omega} - \frac{\kappa}{\omega}\left(\Gamma\right)^{\omega} = -\frac{\eta_1}{\eta_1 - 1}\kappa\left(\Gamma - \psi - \mathbf{h}_{TH,0}\right)^{\omega-1}\mathbf{h}_{TH,0}\ , \tag{4.44}$$

$$\frac{\kappa}{\omega}\left(\Gamma-\psi-\mathbf{h}_{N,0}\right)^{\omega}-\frac{\kappa}{\omega}\left(\Gamma\right)^{\omega}=-\frac{\eta_1}{\eta_1-1}\kappa\left(\Gamma-\psi-\mathbf{h}_{N,0}\right)^{\omega-1}\mathbf{h}_{N,0}\;. \tag{4.45}$$

It can then be seen that[25] the deterministic equilibrium or steady state level of hours worked in each sector is uniquely determined if the participation cost ψ is known. Therefore, ψ is set so as to match a given level of hours worked. This calibration strategy follows Burnside and Eichenbaum (1996). Using their estimates, the deterministic equilibrium hours worked are set to 324.8/1369[26], and Γ is normalised to 1. Burnside and Eichenbaum also found that the choice of ψ does not alter the results. Since the participation cost ψ is the same for both sectors, this calibration strategy forces steady state hours to be the same in the two sectors of the Home country[27]. The same happens in the Foreign country[28].

Moreover, because equations (4.44) and (4.45) have to hold at all dates t, hours always have to stay constant along the solution path. As a result, with this particular choice of preferences, all the adjustment in the labour inputs takes place through the extensive margin, i.e. the participation rates $n_{TH,t}$ and $n_{N,t}$. A second consequence is that, by equations (4.15) to (4.18), the Home relative wage (wage paid in the tradeable sector divided by the wage paid in the nontradeable sector) stays constant at all dates t. The explanation for this is that because individuals can move across sectors and wages are flexible, the relative wage is always kept equal to the marginal rate of substitution between hours in the two sectors, which here is constant and equal to one. The marginal rate of substitution between hours stays constant because hours stay constant along the solution path, and it is equal to one because the hours worked in the two sectors are equal in the steady state. It is important to remember that, even with the equalisation of wages, lotteries and the market for contingent contracts are still needed, in order to be able to write that individuals maximise expected utility ex-ante subject to the expected budget constraint

In practice, the model behaves like a sticky-prices sectoral variant of Hansen's (1985)

[25]Given the function v and the degree of monopolistic competition in the labour market η_1.

[26]The ratio between average hours worked in a year and total hours available, according to Burnside and Eichenbaum's estimates.

[27]As a result, steady state real wages will also be the same in the two Home sectors.

[28]Given that the participation cost is forced to be the same, steady state hours in the Foreign sectors will be the same as in the Home sectors. A different participation cost ψ^* may be introduced, but, in the light of Burnside and Eichenbaum's results, the implications of having $\psi^* \neq \psi$ are not likely to be interesting for the transmission of shocks.

indivisible labour model. In analogy with the literature on indivisible labour models[29], the preference parameter κ could be calibrated with the first order conditions for the labour input. However, κ does not enter the dynamic equations and therefore its calibration is irrelevant.

As explained in Section 4.3, the steady state terms of trade depends on the unconditional means of the productivity processes. These unconditional means may be difficult to estimate at the sectoral level if data is not available, but on the other hand, they do not enter the dynamic log-linearised equations directly. However, in an empirical application of the model it would be desirable to match the tradeable-nontradeable employment ratios in both countries, that is, $\frac{n_{TH}}{n_N}$ and $\frac{n^*_{TF}}{n^*_N}$. Data is almost always available on these employment ratios, and they are important characteristics of the economy. Moreover, it is plausible to assume that in the steady state these ratios are determined by technology and not by preference parameters. Therefore, the determination of the steady state or deterministic equilibrium terms of trade assumes that the ratios $\frac{n_{TH,0}}{n_{N,0}}$ and $\frac{n^*_{TF,0}}{n^*_{N,0}}$ are known, while $\frac{z_{TH,0}}{z_{N,0}}$ and $\frac{z^*_{TF,0}}{z^*_{N,0}}$ are determined endogenously[30]. As stated before, $\frac{z_{TH,0}}{z_{N,0}}$ and $\frac{z^*_{TF,0}}{z^*_{N,0}}$ do not enter directly the dynamic log-linearised equations, but they do so only indirectly, through their impact on the steady state terms of trade. This calibration strategy may be preferable to a simple normalization of $\frac{z_{TH,0}}{z_{N,0}}$ and $\frac{z^*_{TF,0}}{z^*_{N,0}}$ in an empirical application of the model.

4.5 Findings

Having laid out and solved numerically the model with sectors and individual allocative choices, it is possible to investigate the transmission of shocks to the key macroeconomic variables.

In order to be able to do so, it is necessary to select a baseline parametrization, a benchmark that allows the effects of changes in key parameters of the model to be compared. The best benchmark is of course one that does not give more influence to one country or sector over the other one, and that reflects the choices most commonly made in the literature. This baseline parametrization, which could therefore be described as

[29] Both Hansen and Burnside, Eichenbaum and Rebelo in their indivisible labour case use the first order condition with respect to hours or employment in the calibration of their equivalent for the parameter κ.

[30] However, this approach is not viable when ϕ equals one. In this particular case the calibration strategy assumes that $\frac{z_{TH,0}}{z_{N,0}}$ and $\frac{z^*_{TF,0}}{z^*_{N,0}}$ are known, while the ratios $\frac{n_{TH,0}}{n_{N,0}}$ and $\frac{n^*_{TF,0}}{n^*_{N,0}}$ are determined endogenously.

"symmetric" or standard, is reproduced in Table 4.1.

All the exogenous stochastic processes in the model are assumed to be AR(1). Therefore, monetary policy is not specified by means of an interest rate rule. This is because shocks to the interest rate rule are not unambiguously monetary in nature, furthermore, with an interest rate rule it is not possible to isolate the genuine transmission mechanism from the monetary policy reaction function, since the response of all variables depends on the behaviour of the central bank.

The values for σ, φ, φ^* and the autocorrelation coefficients for the nominal growth rates of money in the two countries are taken from Galí (2003). The assumption of log utility for consumption is standard. The fact that the probabilities of not changing prices are set equal to 0.75 implies an average price duration of one year in both countries. Andrés, López-Salido and Vallés (2002) choose one as the value for the elasticity of money demand with respect to consumption, a value taken from the long run estimates of Lucas (1988). This motivates the choice of $\varepsilon = 1$ in this model[31]. The choices of unitary elasticities of substitution, $\phi = \theta = 1$, and of equal weights in the consumption indexes, $\gamma = \delta = \delta^* = 1$, are made to ensure symmetry between countries and sectors. The values for β and η_2 are taken from Pappa (2002). η_1 is set equal to η_2, implying an equal markup (15%) in both the labour and the goods market. The choice of $\alpha = 1$ corresponds to constant marginal labour productivity. An autoregressive parameter for productivity of 0.9 is a standard choice in the real business cycle literature, in line with many empirical estimates. In the baseline parametrization all the exogenous processes for productivity have the same characteristics, therefore the same autoregressive parameter is chosen for tradeables and nontradeables, Home and Foreign. Similarly, a common autoregressive parameter is chosen for government expenditures. The value of 0.9 comes from Rotemberg and Woodford (1995), and it is close to the estimates of Christiano and Eichenbaum (1992).

The consequences of shocks for relative prices and allocations

This section looks at the predictions of the model regarding relative prices and allocations after a shock occurs. These predictions can be better understood by looking at the supply and demand for relative output $\frac{Y_{TH}}{Y_N}$ (that is, the ratio of tradeable to nontradeable output), as functions of the relative price $\frac{P_{TH}}{P_N}$. For simplicity, the analysis is limited to

[31] If $\varepsilon = \sigma = 1$, in the model the elasticity of demand for real money balances with respect to consumption is equal to one.

Table 4.1: Baseline parametrisation

Preferences:		
β	Discount factor	0.99
σ	Risk aversion for consumption	1
ε	Risk aversion for real money balances	1
ϕ	Elasticity of substitution tradeable-nontradeable goods	1
γ	Weight of nontradeable goods in the total consumption index	0.5
θ	Elasticity of substitution Home-Foreign tradeable goods	1
δ	Weight of Foreign goods in the Home tradeable consumption index	0.5
δ^*	Weight of Foreign goods in Foreign tradeable consumption index	0.5
Production:		
η_1	Degree of monopolistic competition in the labour market	7.66
η_2	Degree of monopolistic competition in the goods market	7.66
φ	Probability of not changing prices in the Home country	0.75
φ^*	Probability of not changing prices in the Foreign country	0.75
α	Labour productivity	1
Exogenous processes:		
Autoregressive parameters for nominal money growth		0.5
Autoregressive parameters for productivity		0.9
Autoregressive parameters for government expenditure		0.9

the Home country: all the findings reported from now on can be extended to the Foreign country, which is not left out of the picture, since Foreign shocks may have an impact on the Home economy as well.

What follows is the derivation of the demand and supply for relative output $\frac{Y_{TH}}{Y_N}$ in the model.

The long-run demand and supply for relative output

The supply for relative output describes how firms adjust production $\frac{Y_{TH}}{Y_N}$ following changes in the relative price $\frac{P_{TH}}{P_N}$. This relationship is much easier to obtain using steady state equations. As explained in Section 4.3, the model does not have a unique steady state or deterministic equilibrium, and after a shock the economy may or may not return in the long-run to the initial steady state. The steady state equations hold true across all the possible steady states. By log-linearising the steady state equations around the initial steady state, it is possible to find out the long run implications of the model for the variables of interest.

For any variable X, let X_0 denote the value of the variable in the initial deterministic equilibrium or steady state, and X_{SS} denote the value in the new steady state. Let $\widehat{X} \equiv \frac{X_{SS}-X_0}{X_0} \simeq \log\left(\frac{X_{SS}}{X_0}\right)$ (without subscript) denote the approximate log-deviation from the initial steady state. To find the long-run relationship between relative output and the relative price implied by supply I proceed along the following steps. First, I divide the steady state version of equation (4.1) by (4.2) and I log-linearise the resulting expression. Then I do the same with the first-order conditions that describe the steady state optimal price setting in sectors TH and N. Some algebraic manipulation leads to the following supply relationship that holds across steady states:

$$\widehat{P}_{TH} - \widehat{P}_N = -\frac{1}{\alpha}\left(\widehat{z}_{TH} - \widehat{z}_N\right) + \frac{1-\alpha}{\alpha}\left(\widehat{Y}_{TH} - \widehat{Y}_N\right) .$$

In the case of decreasing marginal labour productivity, $\frac{1-\alpha}{\alpha}$ is positive, thus the relative supply curve is upward-sloping: the supply of relative output increases when the relative price increases. This happens because with decreasing marginal labour productivity the marginal productivity of labour falls with production. The relationship between relative output and relative prices says that if monopolistic competitive firms experience an increase in demand for their good they will charge higher prices to compensate for the fall in productivity. An increase of productivity in the TH sector shifts the relationship down, and an increase of productivity in the N sector shifts the relationship up.

With constant marginal labour productivity ($\alpha = 1$) there is no effect of the relative price $\frac{P_{TH}}{P_N}$ on the supply of $\frac{Y_{TH}}{Y_N}$, thus the relative supply relationship is horizontal. If increasing marginal labour productivity ($\alpha > 1$) was introduced, the slope of the relative supply curve would become negative. The long-run supply relationship with $\alpha < 1$ and $\alpha = 1$ is shown in Figure 11.

The procedure to find the demand relationship is as follows. First, I aggregate and divide the steady state version of (4.40) by its equivalent for sector N, and then take logs. The portion of demand for Y_{TH} that comes from the Foreign country can be substituted out, using the Home and Foreign aggregate resource constraints and the demands (4.33), (4.35) and (4.36), log-linearised around the initial steady state. The relationship between relative output and relative prices implied by demand is:

$$\widehat{Y}_{TH} - \widehat{Y}_N = -\phi\left(\widehat{P}_{TH} - \widehat{P}_N\right) + (1-\phi)(1-k_1)\widehat{T} - (1-k_1)k_4 dB + k_6 dG_{TH} - k_7 dG_N .$$

The equation says that the demand for relative output decreases when the relative price $\frac{P_{TH}}{P_N}$ increases. The relationship is affected by changes in T, B, G_{TH} and G_N across steady states. For example, if government expenditure on tradeable (nontradeable) goods increases, relative output $\frac{Y_{TH}}{Y_N}$ goes up (down). The coefficients $1 - k_1$, k_4, k_6, and k_7 are all positive[32]. The long-run demand relationship is shown in Figure 11.

The response of Home relative output to changes in the terms of trade is ambiguous, it depends on the sign of $(\phi - 1)$. If ϕ is larger than one, the response is negative. This happens because whenever ϕ is larger than one, an increase in the relative price of tradeables to nontradeables, $\frac{P_T}{P_N}$, causes a more than proportional fall in the relative consumption of $\frac{C_T}{C_N}$, that is, consumers substitute a large amount of tradeables with nontradeables. The above equation then says that, keeping $\frac{P_{TH}}{P_N}$, B, G_{TH} and G_N fixed[33], an increase in the terms of trade increases the relative price $\frac{P_T}{P_N}$, and if ϕ is larger than one relative output demand $\frac{Y_{TH}}{Y_N}$ falls, because of the large reallocation of Home consumption towards nontradeables. However, if ϕ is less than one, the Home substitution towards nontradeable goods is less important than the positive effect on Foreign demand for Y_{TH} due to expenditure switching[34].

If dB is greater than zero, the Home country becomes a lender and the Foreign country a borrower in the long run. In other words, there is an increase in wealth in the Home country and a decrease in wealth in the Foreign country. The negative wealth effect in the Foreign country depresses Foreign demand for Y_{TH}, thus explaining the negative relationship between changes in B and changes in $\frac{Y_{TH}}{Y_N}$.

A simple manipulation of the equations for production in the steady state leads to the following relationship between changes in relative employment $\frac{n_{TH}}{n_N}$ and changes in relative output $\frac{Y_{TH}}{Y_N}$:

$$\widehat{n}_{TH} - \widehat{n}_N = \frac{1}{\alpha}\left(\widehat{Y}_{TH} - \widehat{Y}_N\right) - \frac{1}{\alpha}\left(\widehat{z}_{TH} - \widehat{z}_N\right)$$

[32] k_1, k_6, and k_7 are given in Appendix B, while $k_4 = \frac{1}{1-k_1+k_1^*}\left(\frac{1}{\beta} - 1\right)\left(\frac{C_0}{C_{T0}} + \frac{P_{T0}}{e_0 P_{T0}^*}\frac{C_0}{C_{T0}^*}\right)$. Since $B_0 = G_{TH,0} = G_{N,0} = 0$ by assumption, the demand equation is linearised (not log-linearised) with respect to bond holdings and government expenditures. Linear deviations are normalised relative to initial steady state consumption: $dX \equiv \frac{X_{SS}}{C_0}$.

[33] Supply and demand determine $\widehat{Y}_{TH} - \widehat{Y}_N$ and $\widehat{P}_{TH} - \widehat{P}_N$, given $\widehat{z}_{TH} - \widehat{z}_N$, $\widehat{T}$, dB, G_{TH} and G_N. $\widehat{z}_{TH} - \widehat{z}_N$, G_{TH} and G_N are exogenous, $\widehat{T}$ and dB are endogenous variables, determined by other steady state equations.

[34] An increase in the terms of trade brings about an expenditure switching effect, because Foreign tradeable goods are more expensive compared to Home tradeable goods. The expenditure switching effect increases demand and therefore production of Home tradeable goods.

The short-run demand and supply for relative output

For any variable X, let $\widehat{X}_t \equiv \frac{X_t - X_0}{X_0} \simeq \log\left(\frac{X_t}{X_0}\right)$ denote the approximate short-run log-deviation from the initial steady state[35].

The derivation of the forward-looking equation for inflation in the Home tradeable goods sector is shown in Appendix 4.A. By subtracting the forward-looking equations for inflation in the two sectors, it is possible to understand how the short-run relative price adjusts following changes in output:

$$
\begin{aligned}
\widehat{P}_{TH,t} - \widehat{P}_{N,t} = {} & E_t \sum\nolimits_{j=1}^{\infty} (\varphi\beta)^j \left(\pi_{TH,t+j} - \pi_{N,t+j}\right) + \left(\frac{1-\varphi\beta}{1+\frac{1-\alpha}{\alpha}\eta_2}\frac{1-\varphi}{\varphi}\right) \cdot \\
& \cdot E_t \sum\nolimits_{j=0}^{\infty} (\varphi\beta)^j \left(\widehat{MC}_{TH,t+j} - \widehat{MC}_{N,t+j}\right) ,
\end{aligned}
$$

where:

$$
\widehat{MC}_{i,t+j} = \widehat{W}_{i,t+j} - \widehat{P}_{i,t+j} - \frac{1}{\alpha}\widehat{z}_{i,t+j} + \frac{1-\alpha}{\alpha}\widehat{Y}_{i,t+j} \; ; \quad i = TH, \; N..
$$

According to the above equation, short run movements in $\frac{P_{TH,t}}{P_{N,t}}$ reflect expectations of future inflation and real marginal cost differentials[36]. If the inflation rate and the real marginal cost in one sector are equal to those in the other sector, then no changes in the relative price $\frac{P_{TH,t}}{P_{N,t}}$ occur.

It is possible to find also the short-run relationship between relative output and the relative price implied by demand. The procedure is analogous to the one described for the long-run, and it leads to the following relationship[37]:

$$
\begin{aligned}
\widehat{Y}_{TH,t} - \widehat{Y}_{N,t} = {} & -\phi\left(\widehat{P}_{TH,t} - \widehat{P}_{N,t}\right) + (1-\phi)(1-k_1)\widehat{T}_t + \\
& -(1-k_1)\frac{\beta}{1-\beta}k_4\left(\frac{1}{\beta}dB_{t-1} - dB_t\right) + k_6 dG_{TH,t} - k_7 dG_{N,t} \quad (4.46)
\end{aligned}
$$

[35]And let $dX_t \equiv \frac{X_t}{C_0}$ denote normalised linear deviations.

[36]Notice that $\widehat{P}_{TH,t} - \widehat{P}_{N,t}$ appears both on the right and on the left-hand side of the relationship. Of course, it is possible to solve it for the change in the relative price $\frac{P_{TH,t}}{P_{N,t}}$, but in that case the analogy with the long-run supply relationship would be less immediate.

[37]This short-run demand relationship can be solved forward, as it is the short run supply relationship, but in that case the analogy with the long-run demand relationship would be less immediate.

The short-run relationship between changes in relative employment $\frac{n_{TH,t}}{n_{N,t}}$ and relative output $\frac{Y_{TH,t}}{Y_{N,t}}$:

$$\widehat{n}_{TH,t} - \widehat{n}_{N,t} = \frac{1}{\alpha}\left(\widehat{Y}_{TH,t} - \widehat{Y}_{N,t}\right) - \frac{1}{\alpha}\left(\widehat{z}_{TH,t} - \widehat{z}_{N,t}\right)$$

The advantage of the graphical apparatus presented in Figure 11 is that it simplifies the explanation and helps understanding how monetary, productivity and government expenditure shocks affect relative prices and allocations. Shocks shift the relationships both in the short and in the long run, and these shifts can be inferred from the analytical expressions which have been derived. However, it may be worth pointing out that since only temporary, not permanent, shocks are considered, long-run effects on relative prices and allocations are possible only if temporary shocks induce permanent wealth effects, that is, if they cause dB to differ from zero.

Moreover, the equations show that the effects of aggregate shocks can be obtained simply as the sum of sectoral shocks.

The consequences of monetary shocks

Consider a positive monetary shock in the Home country, consisting of a one percent innovation in the Home money growth rate, under the baseline parametrization of Table 4.1.

After having observed the monetary shock, firms in the TH and N sectors expect higher future inflation rates and higher future real marginal costs[38]. However, the two sectors have the same degree of price stickiness, therefore expected future inflation will be the same for TH and N goods, and expected future real marginal costs will also be the same in the two sectors. As a consequence, the relative supply relationship does not shift in the short run after a money shock. Moreover, under the baseline parametrization ϕ is equal to 1 and there are no wealth effects, therefore the relative demand relationship also does not shift in the short run. Since neither the demand nor the supply relationship shift, under the baseline parametrization money shocks are neutral with respect to relative prices and allocations, as shown in the first column of Table 4.2.

[38] Higher future real marginal costs will occur because wages will rise after a monetary shock, but prices will not fully adjust. Moreover, if $\alpha < 1$, the increase in output after a monetary shock will contribute to the rise in real marginal costs.

Table 4.2: Responses to a Home monetary shock

t	Baseline	$\phi = 0.2$	$\phi = 5$	$\sigma = 2$	$\theta = 0.2$	$\theta = 5$	$\theta = 5$ & $\alpha = 0.7$
				Response of $\frac{P_{TH}}{P_N}$			
1	0	0	0	0	0	0	0.057
5	0	0	0	0	0	0	0.119
21	0	0	0	0	0	0	-0.014
				Response of $\frac{Y_{TH}}{Y_N}$			
1	0	0.324	-1.463	0.178	-0.570	2.848	2.777
5	0	0.122	-0.456	0.052	-0.167	0.834	0.872
21	0	0.028	0.015	-0.007	0.021	-0.106	-0.097
				Response of $\frac{n_{TH}}{n_N}$			
1	0	0.324	-1.463	0.178	-0.570	2.848	3.968
5	0	0.122	-0.456	0.052	-0.167	0.834	1.246
21	0	0.028	0.015	-0.007	0.021	-0.106	-0.138

Note: The baseline parametrization is shown in Table 4.1. Responses are percent deviations from steady state values, t is measured in quarters. The other specifications differ from the baseline only with respect to the parameters indicated at the top of each column.

However, monetary shocks may induce wealth and terms of trade effects that shift the relative demand relationship. In these situations monetary shocks are not neutral. For example, consider the case (shown in the second column of Table 4.2) of ϕ being less than one, while the remaining parameters are left to the values of the benchmark parametrisation in Table 4.1. A positive monetary shock causes an increase (deterioration) of the terms of trade[39], thus, as equation (4.46) shows, the relative demand curve shifts to the right. The second column of Table 4.2 shows that, as a result of the shift due to the terms of trade effect, relative output $\frac{Y_{TH}}{Y_N}$ increases initially. This happens because the terms of trade depreciation makes Home tradeables more attractive and substitution towards nontradeables C_N is low ($\phi < 1$). In addition to this terms of trade effect, there is also a wealth effect. The latter arises because the positive monetary shock makes tradeables goods C_T more expensive, and since substitution towards nontradeables C_N is low, Home individuals prefer to borrow in order to be able to afford tradeables C_T in the current period[40]. The wealth effect influences the relative demand (4.46) alongside

[39]Under all parametrisations, a positive monetary shock causes a depreciation of the Home currency. Since prices are rigid, this always results in a deterioration of the terms of trade T_t, and in an increase in the price of tradeables $P_{T,t}$.

[40]If $\phi > 1$, Home individuals would be more willing to postpone the consumption of tradeables C_T, so

the terms of trade effect, but the wealth effect is gradual, since B is a stock variable that adjusts slowly, while the terms of trade effect is maximum at impact[41]. Both the wealth effect and the terms of trade effect are permanent, and in the long run there is a higher demand for relative output $\frac{Y_{TH}}{Y_N}$.

The third column of Table 4.2 shows the case of ϕ greater than one. After a positive Home money shock, the relative demand curve shifts initially to the left, then gradually to the right. As a result, an increase in Home money supply increases at impact relative output and employment if ϕ is less than one, and decreases at impact relative output and employment if ϕ is greater than one.

Similar reasonings help explain the remaining columns of Table 4.2. Shifts in the relative demand relationship occur if either ϕ, θ or σ are different from one. In the presence of decreasing marginal productivity of labour, monetary shocks affect relative prices as well, since the relative supply curve is no longer horizontal. For example, if $\theta = 5$ and $\alpha = 0.7$, then a positive Home monetary shock causes initially an increase in relative prices, followed by a decrease.

The consequences of productivity shocks

Let us consider the two types of productivity shocks in the Home country, consisting of one percent innovations in the productivity processes for tradeables and nontradeables.

Positive productivity shocks lower real marginal costs for firms, thus inducing them to lower their prices. Therefore, after a productivity shock in one of the two sectors is observed, both the expected future inflation differential and the expected future real marginal cost differential change, and as a result the short run relative supply relationship shifts. In particular, the short run relative supply relationship shifts up after a positive productivity shock in the TH sector, down after a positive productivity shock in the N sector. As for the relative demand relationship, it does not shift under the baseline parametrization, since in this particular case there are no permanent wealth effects, and changes in the terms of trade are not transmitted whenever ϕ is equal to 1.

The columns of Tables 4.3 and 4.4 can be understood then by simply looking at Figure 11 and "introducing the dynamics" to it. For example (Table 4.3), a positive productivity

they would lend resources abroad.

[41] Since wealth effects (changes in B) constitute a reallocation of wealth across countries, they are always associated with permanent terms of trade effects. When there are no wealth effects, in the long run the terms of trade goes back to its initial value.

Table 4.3: Responses to a Home technology shock in tradeables

t	Baseline	$\alpha = 0.85$	$\alpha = 0.7$	$\phi = 5$	$\theta = 5$	$\varphi = 0.9$	AR $= 0$
				Response of $\frac{P_{TH}}{P_N}$			
1	-0.194	-0.129	-0.099	-0.194	-0.194	-0.055	-0.064
5	-0.457	-0.351	-0.291	-0.457	-0.457	-0.181	-0.020
21	-0.138	-0.147	-0.150	-0.138	-0.138	-0.140	0
				Response of $\frac{Y_{TH}}{Y_N}$			
1	0.194	0.129	0.099	0.582	0.444	0.055	0.064
5	0.457	0.351	0.291	1.371	1.232	0.181	0.020
21	0.138	0.147	0.150	0.415	0.277	0.140	0
				Response of $\frac{n_{TH}}{n_N}$			
1	-0.806	-1.025	-1.286	-0.418	-0.556	-0.945	-0.936
5	-0.199	-0.359	-0.522	0.715	0.576	-0.476	0.020
21	0.017	0.030	0.040	0.294	0.156	0.019	0

Note: See Table 4.2.

shocks in the TH sector shifts the relative supply down, thus lowering relative prices and increasing relative output. Moreover, since now tradeable output can be produced with less labour, relative employment falls at impact. All these responses are temporary, and they are opposite in sign if the positive productivity shock occurs in the N sector (Table 4.4).

In the presence of decreasing marginal productivity of labour (columns 3 and 4 of both tables), the effects of productivity shocks on relative prices and output are analogous in sign, but their magnitude at impact is reduced by the positive slope in the relative supply curve. Moreover, with decreasing marginal productivity of labour, the response of relative employment after a productivity shock is amplified. This happens because a positive productivity shock in one sector allows firms within that sector to dispense with labour, and the less productive are the workers, the more labour is dispensed.

The response of relative output can be considerably higher if the parameters ϕ or θ are different from one. This happens because when ϕ and θ are different from one productivity shocks bring about permanent wealth effects, and as a result the relative demand relationship (whose slope is given by the inverse of ϕ) shifts.

The last columns of Tables 3 and 4 show that both the parameter φ and the autoregressive parameter of the productivity shock affect affect the size of and the persistence of the responses. This happens because these parameters determine the expected future

Table 4.4: Responses to a Home technology shock in nontradeables

t	Baseline	$\alpha = 0.85$	$\alpha = 0.7$	$\phi = 5$	$\theta = 5$	$\varphi = 0.9$	AR $= 0$
	Response of $\frac{P_{TH}}{P_N}$						
1	0.194	0.129	0.099	0.194	0.194	0.055	0.064
5	0.457	0.351	0.291	0.457	0.457	0.181	0.020
21	0.138	0.147	0.150	0.138	0.138	0.140	0
	Response of $\frac{Y_{TH}}{Y_N}$						
1	-0.194	-0.129	-0.099	-0.753	-0.194	-0.055	-0.064
5	-0.457	-0.351	-0.291	-1.805	-0.457	-0.181	-0.020
21	-0.138	-0.147	-0.150	-0.531	-0.138	-0.140	0
	Response of $\frac{n_{TH}}{n_N}$						
1	0.806	1.025	1.286	0.247	0.806	0.945	0.936
5	0.199	0.359	0.522	-1.148	0.199	0.476	-0.020
21	-0.017	-0.030	-0.040	-0.409	-0.017	-0.019	0

Note: See Table 4.2.

inflation and real marginal cost differentials.

The consequences of government expenditure shocks

Under the baseline parametrization, increases in government expenditures on either good do not shift the relative supply relationship in the short-run, since the expected future inflation differential and the expected future real marginal cost differential are not affected. However, as shown by the analytical expressions for the short and the long run, the relative demand relationship is shifted by changes in government expenditure. Therefore, when the relative supply relationship is horizontal, government expenditure shocks affect only relative quantities, but not prices. When the relative supply relationship is upward-sloping (decreasing marginal productivity of labour), government expenditure shocks affect both relative quantities and prices.

For example, after a positive government expenditure shock in tradeables (nontradeables) the demand relationship in Figure 11 shifts up (down) in the short run, and, as a result, relative output and employment increase (decrease), in order to accommodate the changes in demand brought about by government spending. In the case of decreasing marginal productivity of labour, relative prices must increase (decrease), and the effects on relative employment are more pronounced, since increases (decreases) in output must be accompanied by higher increases (decreases) in employment, to compensate for the

Table 4.5: Responses to a Home government expenditure shock in tradeables

t	Baseline	$\alpha = 0.85$	$\gamma = 0.25$	$\gamma = 0.75$	$\frac{z_{TH,0}}{z^*_{TF,0}} = 5$	$\frac{z_{TH,0}}{z^*_{TF,0}} = 0.2$
			Response of $\frac{P_{TH}}{P_N}$			
1	0	0.014	0	0	0	0
5	0	0.037	0	0	0	0
21	0	0.016	0	0	0	0
			Response of $\frac{Y_{TH}}{Y_N}$			
1	0.707	0.693	0.452	1.917	0.473	1.057
5	0.464	0.427	0.296	1.258	0.310	0.694
21	0.086	0.070	0.055	0.233	0.057	0.129
			Response of $\frac{n_{TH}}{n_N}$			
1	0.707	0.816	0.452	1.917	0.473	1.057
5	0.464	0.502	0.296	1.258	0.310	0.694
21	0.086	0.083	0.055	0.233	0.057	0.129

Note: See Table 4.2.

decreasing productivity.

Tradeables and nontradeables government expenditure shocks have a symmetric impact on relative output, unless the weight of nontradeable goods in total consumption γ, and the weights of Foreign tradeables δ and δ^* (not shown in the Tables), are different from 0.5. The parameters γ, δ and δ^* affect the coefficients k_1, k_6 and k_7 which appears in the relative demand equations. This happens because γ, δ and δ^* affect the steady state terms of trade, and in this way they affect the coefficients of the approximated log-linearised solution.

Moreover, as explained in Section 4.4, the steady state terms of trade is also affected by the values chosen for the unconditional means of the productivity processes. Consequently, different assumptions on the ratio between the unconditional means of the tradeable productivity shocks in the two countries also affect the coefficients of the relative demand relationships. As an example, the last columns of Tables 4.5 and 4.6 show that different assumptions on these ratios affect the responses of relative output and prices to government expenditure shock. In other words, what these columns show is that a wrong calibration of the steady state terms of trade may introduce erroneous responses.

Table 4.6: Responses to a Home government expenditure shock in nontradeables

t	Baseline	$\alpha = 0.85$	$\gamma = 0.25$	$\gamma = 0.75$	$\frac{z_{TH,0}}{z^*_{TF,0}} = 5$	$\frac{z_{TH,0}}{z^*_{TF,0}} = 0.2$
			Response of $\frac{P_{TH}}{P_N}$			
1	0	-0.014	0	0	0	0
5	0	-0.037	0	0	0	0
21	0	-0.016	0	0	0	0
			Response of $\frac{Y_{TH}}{Y_N}$			
1	-0.707	-0.693	-1.355	-0.639	-0.473	-1.057
5	-0.464	-0.427	-0.889	-0.419	-0.310	-0.694
21	-0.086	-0.070	-0.165	-0.078	-0.057	-0.129
			Response of $\frac{n_{TH}}{n_N}$			
1	-0.707	-0.816	-1.355	-0.639	-0.473	-1.057
5	-0.464	-0.502	-0.889	-0.419	-0.310	-0.694
21	-0.086	-0.083	-0.165	-0.078	-0.057	-0.129

Note: See Table 4.2.

4.6 Conclusion

The explicit consideration of relative or allocative effects of exogenous shocks in new open economy models is potentially very beneficial, not only because it allows us to understand the models better, but also because it provides an additional dimension to test or, alternatively, calibrate the models. The analysis of sectoral shocks has probably been limited so far by the availability of disaggregated statistics, but as more data becomes available this limitation is likely to disappear. In an open economy setting, the distinction between tradeables and nontradeables is then the natural point to start.

Naturally, the results of the analysis crucially depend on the modelling choices (for example, home bias, substitutability among goods, and so on). To mitigate (but not completely eliminate)[42] this problem, the approach taken here is to choose general functional forms where possible, and conduct detailed sensitivity analysis.

The core message of the chapter is that not only sector-specific supply or demand shocks have small to sizeable relative effects, but so do aggregate monetary shocks. This finding is particularly interesting because the empirical literature has consistently pointed out that money has heterogeneous effects. These may be explained, as the Introduction suggests, by characteristics such as the degree of tradability at the sectoral level.

[42] Other modelling options that have not been taken explicitly in consideration are market segmentation or local currency pricing.

Llaudes finds that a contractionary monetary policy shock causes a medium-run decline in tradeable sector output, while output in the nontradeable sector tends to increase after the shock. I obtain in my model analogous results by setting either $\phi < 1$ or $\theta > 1$. Notably, Llaudes' results are robust across countries, but they are of difficult interpretation, since in some countries both the nominal and the real exchange rates appreciate[43] after the contractionary monetary policy shock, while in other countries[44] there is a depreciation. This "exchange rate puzzle" is not peculiar to Llaudes' paper, but apparently it is a feature of other empirical studies. Since the source of changes in the relative price is the exchange rate, because of the exchange rate puzzle it is problematic to interpret the same effect on the tradable and nontradable output both under an appreciation and under a depreciation. However, one of the advantages of the model that I put forward is that it can reproduce short-run responses that are different in sign from the medium-run responses.

There is perhaps another reason to study the allocative effects of monetary or fiscal shocks. In the new open economy framework it is possible to devise optimal policies, that is, policies aimed at reducing the welfare losses brought about by fluctuations, in the presence of market imperfections and nominal rigidities. For the moment, the literature has not focused on the allocative effects of monetary or fiscal policies, but it makes sense also to do so, especially when there are costs of moving resources across sectors. This interesting issue is left to future research.

[43] Italy, France, and the U.K..

[44] Germany and Japan.

Appendix 4.A: Optimal price setting

This appendix shows how to derive the log-linearised equation (4.54) from the problem of firm f_{TH} changing its price at time t. As shown in Section 4.2, the first-order condition describing optimal price setting is:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{1}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH})(1-\eta_2) + \\ +\eta_2 \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y_{TH,t+j|t}(f_{TH})} \cdot \frac{y_{TH,t+j|t}(f_{TH})}{p_{TH,t}(f_{TH})} \end{array} \right] = 0 \, .$$

Given the sequences $\{C_t\}$, $\{P_t\}$ and $\{W_{TH,t}\}$, the sequences of shocks and the initial conditions, each producer that chooses a new price in period t will choose the same price $p_{TH,t}(f_{TH})$ and the same level of output $y_{TH,t+j|t}(f_{TH})$. Then the optimal paths of prices $\{p_{TH,t}(f_{TH}), P_{TH,t}\}$ satisfy the above first-order condition and the following law of motion:

$$P_{TH,t} = \left[\varphi P_{TH,t-1}^{1-\eta_2} + (1-\varphi)\, p_{TH,t}(f_{TH})^{1-\eta_2}\right]^{\frac{1}{1-\eta_2}} .$$

By log-linearising the law of motion above I get:

$$\widehat{X}_t = \frac{\varphi}{1-\varphi}\pi_{TH,t} \, ,$$

where $X_t \equiv \frac{p_{TH,t}(f_{TH})}{P_{TH,t}}$, and $\pi_{TH,t} \equiv \log \frac{P_{TH,t}}{P_{TH,t-1}}$ is the inflation rate in the Home tradeable goods sector. Note that:

$$\widehat{X}_{t+j} = \widehat{X}_t - \sum_{s=1}^{j} \pi_{TH,t+s} = \frac{\varphi}{1-\varphi}\pi_{TH,t} - \sum_{s=1}^{j} \pi_{TH,t+s} \, ,$$

where $X_{t+j} \equiv \frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}}$. From equation (4.1) I obtain:

$$\frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y_{TH,t+j|t}(f_{TH})} = \frac{1}{\alpha} \cdot (z_{TH,t+j})^{-\frac{1}{\alpha}} \cdot \left(y_{TH,t+j|t}(f_{TH})\right)^{\frac{1}{\alpha}-1} .$$

Substituting the above expression into the first-order condition and multiplying by $p_{TH,t}(f_{TH})$ I obtain:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}} \cdot \frac{P_{TH,t+j}}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH})(1-\eta_2) + \\ +\frac{\eta_2}{\alpha} \cdot (z_{TH,t+j})^{-\frac{1}{\alpha}} \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot (y_{TH,t+j|t}(f_{TH}))^{\frac{1}{\alpha}} \end{array} \right] = 0 \,.$$

Now I log-linearise around a deterministic equilibrium in which all the exogenous stochastic processes are set equal to their unconditional means, their variances are set to zero, the individuals hold no internationally traded bond, and all government expenditures are equal to zero. In this deterministic equilibrium $p_{TH,t}(f_{TH}) = P_{TH,t+j}$. I obtain:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j \left[\begin{array}{c} \widehat{X}_{t+j} + \widehat{P}_{TH,t+j} - \widehat{P}_{t+j} + \widehat{y}_{TH,t+j|t}(f_{TH}) + \\ -\left(-\frac{1}{\alpha} \cdot \widehat{z}_{TH,t+j} + \widehat{W}_{TH,t+j} - \widehat{P}_{t+j} + \frac{1}{\alpha} \cdot \widehat{y}_{TH,t+j|t}(f_{TH})\right) \end{array} \right] = 0 \,.$$

Log-linearising equation (4.40) I obtain:

$$\widehat{y}_{TH,t+j|t}(f_{TH}) = -\eta_2 \cdot \widehat{X}_{t+j} + k_1 \widehat{C}_{TH,t+j} + (1-k_1)\,\widehat{C}^*_{TH,t+j} + k_6 dG_{TH,t+j} \,.$$

The coefficients k_1 and k_6 are derived from steady state equations and are given in Appendix 4.B. I can substitute into the log-linearised first-order condition the expressions for $\widehat{X}_{t+j}$ and $\widehat{y}_{TH,t+j|t}(f_{TH})$, and after some simplifications I obtain:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j \left[\begin{array}{c} \left(1+\frac{1-\alpha}{\alpha}\eta_2\right) \frac{\varphi}{1-\varphi}\pi_{TH,t} - \left(1+\frac{1-\alpha}{\alpha}\eta_2\right) \sum_{s=1}^{j} \pi_{TH,t+s} + \widehat{P}_{TH,t+j} - \widehat{W}_{TH,t+j} \\ +\frac{1}{\alpha}\widehat{z}_{TH,t+j} - \frac{1-\alpha}{\alpha} k_1 \widehat{C}_{TH,t+j} - \frac{1-\alpha}{\alpha}(1-k_1)\,\widehat{C}^*_{TH,t+j} - \frac{1-\alpha}{\alpha} k_6 dG_{TH,t+j} \end{array} \right] = 0 \,,$$

which can be further simplified as follows:

$$\begin{gathered} \frac{1}{1-\varphi\beta}\frac{\varphi}{1-\varphi}\left(1+\frac{1-\alpha}{\alpha}\eta_2\right)\pi_{TH,t} = \\ \frac{1}{1-\varphi\beta}\left(1+\frac{1-\alpha}{\alpha}\eta_2\right) E_t \sum_{j=1}^{\infty} (\varphi\beta)^j \pi_{TH,t+j} + \\ -E_t \sum_{j=0}^{\infty} (\varphi\beta)^j \left[\begin{array}{c} \widehat{P}_{TH,t+j} - \widehat{W}_{TH,t+j} + \frac{1}{\alpha}\widehat{z}_{TH,t+j} - \frac{1-\alpha}{\alpha} k_1 \widehat{C}_{TH,t+j} \\ -\frac{1-\alpha}{\alpha}(1-k_1)\,\widehat{C}^*_{TH,t+j} - \frac{1-\alpha}{\alpha} k_6 dG_{TH,t+j} \end{array} \right] . \end{gathered}$$

Finally, simplifying and using the law of iterated expectations, I can write the forward-looking equation for inflation in the Home tradeable goods sector:

$$\pi_{TH,t} = \beta E_t \pi_{TH,t+1} + \left(\frac{1-\varphi\beta}{1+\frac{1-\alpha}{\alpha}\eta_2} \frac{1-\varphi}{\varphi} \right) \left(\begin{array}{c} \widehat{W}_{TH,t} - \widehat{P}_{TH,t} - \frac{1}{\alpha}\widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha} k_1 \widehat{C}_{TH,t} + \\ + \frac{1-\alpha}{\alpha}(1-k_1)\widehat{C}^*_{TH,t} + \frac{1-\alpha}{\alpha} k_6 dG_{TH,t} \end{array} \right) ,$$

which can be rewritten as:

$$\pi_{TH,t} = \beta E_t \pi_{TH,t+1} + \left(\frac{1-\varphi\beta}{1+\frac{1-\alpha}{\alpha}\eta_2} \frac{1-\varphi}{\varphi} \right) \left(\widehat{W}_{TH,t} - \widehat{P}_{TH,t} - \frac{1}{\alpha}\widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha}\widehat{Y}_{TH,t} \right) .$$

The forward-looking equation links current inflation to expected future inflation and real marginal costs, since:

$$\widehat{MC}_{TH,t} = \widehat{W}_{TH,t} - \widehat{P}_{TH,t} - \frac{1}{\alpha}\widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha}\widehat{Y}_{TH,t} ,$$

where $MC_{TH,t}$ is real marginal cost in sector TH. If marginal labour productivity to scale is constant ($\alpha = 1$), the level of output does not affect real marginal costs, and the equation becomes more standard:

$$\pi_{TH,t} = \beta E_t \pi_{TH,t+1} + (1-\varphi\beta)\frac{1-\varphi}{\varphi}\left(\widehat{W}_{TH,t} - \widehat{P}_{TH,t} - \widehat{z}_{TH,t} \right) .$$

Appendix 4.B: The log-linearised model equations

The subscript 0 indicates a variable in the initial deterministic equilibrium or steady state. In what follows, variables with a hat denote log-deviations from the initial deterministic equilibrium or steady state, for example $\widehat{C}_t \equiv \log C_t - \log C_0$. Some equations are mixed log-linear and linear approximations, because they have been linearised with respect to those variables whose initial steady state value is zero by assumption. Linear deviations, prefixed with d, are normalised with respect to consumption, for example $dB_t \equiv \frac{B_t - 0}{C_0}$. The coefficients k_1, k_2, ... are derived from the steady state equations and are given at the end.

Home resource constraint

$$\begin{aligned} dB_t &= \frac{1}{\beta} dB_{t-1} - (1-k_1)\, k_2 k_3 \widehat{T}_t + k_2 k_3 \widehat{Y}_{TH,t} + (1-k_2)\, k_3 \widehat{Y}_{N,t} - k_3 \widehat{C}_t + \\ & \quad - k_2 k_3 k_6 dG_{TH,t} - (1-k_2)\, k_3 k_7 dG_{N,t} \ . \end{aligned} \tag{4.47}$$

To obtain (4.47), aggregate the budget constraint equation (4.9), consolidate it with (4.19), log-linearise and linearise around the steady state and substitute out relative price level changes. $Y_{TH,t} \equiv \left[\int_0^1 y_{TH,t} (f_{TH})^{\frac{\eta_2 - 1}{\eta_2}} df_{TH} \right]^{\frac{\eta_2}{\eta_2 - 1}}$ and $Y_{N,t} \equiv \left[\int_0^1 y_{N,t} (f_N)^{\frac{\eta_2 - 1}{\eta_2}} df_N \right]^{\frac{\eta_2}{\eta_2 - 1}}$ are aggregates for output in the two sectors. $dG_{TH,t} \equiv \frac{G_{TH,t}}{C_0}$ and $dG_{N,t} \equiv \frac{G_{N,t}}{C_0}$ are linear deviations, normalised with respect to steady state consumption. Since $B_t^* = -B_t$ at all dates, it is not necessary to add the Foreign resource constraint to the system of equations.

Nominal interest rates

$$\widehat{i}_t = \left(\frac{1}{1-\beta} \right) \left[(1-k_1) \left(E_t \widehat{T}_{t+1} - \widehat{T}_t \right) + E_t \pi_{TH,t+1} \right] + \widehat{r}_t \ , \tag{4.48}$$

$$\widehat{i}_t^* = \widehat{i}_t + \left(\frac{1}{1-\beta} \right) (\widehat{e}_t - E_t \widehat{e}_{t+1}) \ . \tag{4.49}$$

(4.48) and (4.49) are obtained by log-linearising (4.11) and (4.12), and substituting out price level changes. I can see from (4.49) that the uncovered interest parity condition holds. Variables denoted with π are inflation rates in the four sectors, for example $\pi_{TH,t+1} \equiv$

$\log \frac{P_{TH,t+1}}{P_{TH,t}}$.

Euler equations

$$\begin{aligned}\sigma E_t\widehat{C}_{t+1} - \sigma\widehat{C}_t &= (1-\beta)\widehat{r}_t + (1-k_1)(1-k_2)\left(E_t\widehat{T}_{t+1} - \widehat{T}_t\right) + \\ &+ (1-k_2)E_t\pi_{TH,t+1} - (1-k_2)E_t\pi_{N,t+1}\ , \end{aligned} \tag{4.50}$$

$$\begin{aligned}\sigma E_t\widehat{C}^*_{t+1} - \sigma\widehat{C}^*_t &= (1-\beta)\widehat{r}_t + (k_1^*k_2^* - k_1)\left(E_t\widehat{T}_{t+1} - \widehat{T}_t\right) + \\ &+ (1-k_2^*)\left(E_t\pi^*_{TF,t+1} - E_t\pi^*_{N,t+1}\right)\ . \end{aligned} \tag{4.51}$$

(4.50) and (4.51) are obtained by aggregating and log-linearising (4.13) and its Foreign counterpart, and substituting out relative price level changes.

Money demand equations

$$\varepsilon\widehat{m}_t = \frac{1}{1-\beta}\left[\begin{array}{c}\sigma\widehat{C}_t - \sigma\beta E_t\widehat{C}_{t+1} - \beta(1-k_1)k_2\left(E_t\widehat{T}_{t+1} - \widehat{T}_t\right) + \\ -k_2\beta E_t\pi_{TH,t+1} - (1-k_2)\beta E_t\pi_{N,t+1}\end{array}\right], \tag{4.52}$$

$$\varepsilon\widehat{m}^*_t = \frac{1}{1-\beta}\left[\begin{array}{c}\sigma\widehat{C}^*_t - \sigma\beta E_t\widehat{C}^*_{t+1} + \beta k_1^*k_2^*\left(E_t\widehat{T}_{t+1} - \widehat{T}_t\right) + \\ -k_2^*\beta E_t\pi^*_{TF,t+1} - (1-k_2^*)\beta E_t\pi^*_{N,t+1}\end{array}\right]. \tag{4.53}$$

(4.52) and (4.53) are obtained by aggregating and log-linearising (4.14) and its Foreign counterpart, and substituting out price level changes. $m_t \equiv \frac{M_t}{P_t}$ and $m_t^* \equiv \frac{M_t^*}{P_t^*}$ are real money balances in the two countries.

Forward-looking equations for inflation

$$\pi_{TH,t} = \beta E_t\pi_{TH,t+1} + \left(\frac{1-\varphi\beta}{1+\eta_2\frac{1-\alpha}{\alpha}}\frac{1-\varphi}{\varphi}\right)\left(\begin{array}{c}\widehat{W}_{TH,t} - \widehat{P}_{TH,t} - \frac{1}{\alpha}\cdot\widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha}k_1\widehat{C}_{TH,t} \\ +\frac{1-\alpha}{\alpha}(1-k_1)\widehat{C}^*_{TH,t} + \frac{1-\alpha}{\alpha}k_6 dG_{TH,t}\end{array}\right), \tag{4.54}$$

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + \left(\frac{1-\varphi\beta}{1+\eta_2 \frac{1-\alpha}{\alpha}} \frac{1-\varphi}{\varphi} \right) \left(\widehat{W}_{N,t} - \widehat{P}_{N,t} - \frac{1}{\alpha} \cdot \widehat{z}_{N,t} + \frac{1-\alpha}{\alpha} \widehat{C}_{N,t} + \frac{1-\alpha}{\alpha} k_7 dG_{N,t} \right) , \tag{4.55}$$

$$\pi^*_{TF,t} = \beta E_t \pi^*_{TF,t+1} + \left(\frac{1-\varphi^*\beta}{1+\eta_2 \frac{1-\alpha}{\alpha}} \frac{1-\varphi^*}{\varphi^*} \right) \left(\begin{array}{c} \widehat{W}^*_{TF,t} - \widehat{P}^*_{TF,t} - \frac{1}{\alpha} \cdot \widehat{z}^*_{TF,t} + \frac{1-\alpha}{\alpha} k_1^* \widehat{C}_{TF,t} \\ + \frac{1-\alpha}{\alpha} (1-k_1^*) \widehat{C}^*_{TF,t} + \frac{1-\alpha}{\alpha} k_6^* dG^*_{TF,t} \end{array} \right) , \tag{4.56}$$

$$\pi^*_{N,t} = \beta E_t \pi^*_{N,t+1} + \left(\frac{1-\varphi^*\beta}{1+\eta_2 \frac{1-\alpha}{\alpha}} \frac{1-\varphi^*}{\varphi^*} \right) \left(\widehat{W}^*_{N,t} - \widehat{P}^*_{N,t} - \frac{1}{\alpha} \cdot \widehat{z}^*_{N,t} + \frac{1-\alpha}{\alpha} \widehat{C}^*_{N,t} + \frac{1-\alpha}{\alpha} k_7^* dG^*_{N,t} \right) . \tag{4.57}$$

The derivation of equation (4.54) is shown in Appendix 4.A, and equations (4.55), (4.56) and (4.57) are derived following an analogous procedure. $dG^*_{TF,t} \equiv \frac{G^*_{TF,t}}{C_0^*}$ and $dG^*_{N,t} \equiv \frac{G^*_{N,t}}{C_0^*}$.

Real money balances

$$\widehat{m}_t - \widehat{m}_{t-1} = \widehat{\mu}_t - (1-k_1) k_2 \left(\widehat{T}_t - \widehat{T}_{t-1} \right) - k_2 \pi_{TH,t} - (1-k_2) \pi_{N,t} , \tag{4.58}$$

$$\widehat{m}^*_t - \widehat{m}^*_{t-1} = \widehat{\mu}^*_t + k_1^* k_2^* \left(\widehat{T}_t - \widehat{T}_{t-1} \right) - k_2^* \pi^*_{TF,t} - (1-k_2^*) \pi^*_{N,t} . \tag{4.59}$$

(4.58) and (4.59) are derived from the real money balances identities, substituting out relative price level changes. $\widehat{\mu}_t \equiv \log \frac{M_t}{M_{t-1}}$ and $\widehat{\mu}^*_t \equiv \log \frac{M^*_t}{M^*_{t-1}}$ are nominal money growth rates.

The other equations of the dynamic system are obtained by aggregating and log-linearising equations (4.1), (4.2), (4.3), (4.4) and their Foreign counterparts, equations (4.15), (4.16), (4.17), (4.18) and their Foreign counterparts, equations (4.28) and (4.29), equations (4.30) to (4.39) and their Foreign counterparts[45], and equations (4.40) and (4.42)

[45]The log-linearised equations (4.30) to (4.38) and their Foreign counterparts are also first-differenced in order to eliminate price level changes from the system. Price level changes are substituted with inflation rates, which have no long-run growth.

plus their counterparts. $\widehat{\mu}_t$, $\widehat{\mu}_t^*$, $dG_{TH,t}$, $dG_{N,t}$, $dG_{TF,t}^*$, $dG_{N,t}^*$, $\widehat{z}_{TH,t}$, $\widehat{z}_{N,t}$, $\widehat{z}_{TF,t}^*$, $\widehat{z}_{N,t}^*$ are all exogenous stochastic processes.

The procedure to find the steady state terms of trade, T_0, is described in Section 4.3. After T_0 is found, the coefficients of the log-linearised model are derived from the steady state equations as follows:

$$
\begin{aligned}
k_1 &= \frac{C_{TH0}}{Y_{TH0}}, & k_1^* &= \frac{C_{TF0}}{Y_{TF0}^*}; \\
k_2 &= \frac{P_{T0}C_{T0}}{P_0 C_0}, & k_2^* &= \frac{P_{T0}^* C_{T0}^*}{P_0^* C_0^*}; \\
k_3 &= \frac{P_0}{P_{T0}}, & k_3^* &= \frac{P_0^*}{P_{T0}^*}; \\
k_6 &= \frac{C_0}{Y_{TH0}}, & k_6^* &= \frac{C_0^*}{Y_{TF0}^*}; \\
k_7 &= \frac{C_0}{Y_{N0}}, & k_7^* &= \frac{C_0^*}{Y_{N0}^*}.
\end{aligned}
$$

Chapter 5

The International Transmission of Shocks

5.1 Introduction

This chapter investigates the domestic and the international transmission of monetary, government expenditure and productivity shocks in a dynamic general equilibrium model with nominal rigidities. This will be done primarily with the model presented in the previous chapter, which is characterized by the assumption that individuals cannot work in both sectors, but they can choose both allocations and hours of work. However, in order to generate a better understanding of the transmission, results are also shown for another version of the model, in which individuals can supply hours of work to both sectors contemporaneously, but they cannot choose allocations.

Although the benefits of understanding the international transmission are numerous[1], concerning the general framework of the new open economy models, no comprehensive study exists on a wide range of shocks, nor on the robustness to changes in parameter values. It is fair to say that the international transmission of shocks depends on the modelling strategy chosen. For decades, the Mundell-Fleming-Dornbusch model[2] has provided the framework to study the international transmission of monetary shocks, but now the framework more widely used is the *Redux* (together with its extensions), which has been introduced relatively recently. It is rather more difficult to understand the transmission of

[1] For example, it could provide the empirical VAR literature the restrictions needed to identify and measure the shocks that are in the data.

[2] Mundell, (1962, 1963); Fleming, (1962); Dornbusch, (1976).

shocks in an intertemporal general equilibrium model than in the IS-LM-based Mundell-Fleming-Dornbusch model. Moreover, the literature so far has privileged monetary shocks instead of analysing all shocks.

One of the contributions of this chapter therefore consists in extending the basic model by adding productivity and government expenditure shocks. However, in the open economy shocks can have very different effects according to whether they occur in the tradeable or in the nontradeable sector. Therefore, productivity and government expenditure shocks are distinct for tradeables and nontradeables, but the effects of aggregate shocks of the same nature can be analysed easily. Different assumptions about the model's parameters are considered, and extensive sensitivity analysis is conducted.

An analysis of the transmission mechanism of monetary shocks in a new open economy model can be found in Obstfeld and Rogoff's original *Redux* paper[3], or in Mark (2001). However, the models presented there do not have the same degree of complexity as the more recent models. Some crucial assumptions and their effect on the transmission of money shocks have been analysed separately in recent papers. For example, Tille (2001) analyses the role of different cross-country and within-country substitutability of goods, Hau (2000) analyses the role of the nontraded goods sector, Warnock (2003) covers home bias, and Betts and Devereux (1996, 2000) introduced and analysed the role of local currency pricing. This chapter constitutes a more comprehensive analysis than these studies because: 1) many of the critical parameters already studied in isolation are present, so their role can be analysed simultaneously; 2) the analysis is not confined to monetary shocks; 3) shocks are disaggregated by sector; 4) some novel assumptions are introduced (the assumptions that the marginal productivity of labour may be decreasing, and that individuals cannot work in both sectors). These latter modelling innovations are introduced in a way that preserves comparability with the other papers: constant marginal productivity of labour is treated as a special case, and results are also shown for the case in which individuals can work in both sectors.

The main results of the chapter are as follows. In the short run, monetary shocks dominate the responses of consumption, real and nominal exchange rates, real output and employment. Even without capital in the model, the effects of productivity shocks are quite persistent, and this persistence is due to the nominal rigidities[4]. Government

[3] Obstfeld and Rogoff (1995, also 1996).

[4] The explanation is given on page 154.

expenditure shocks have little impact on the main variables, and often no impact at all. The propagation of Foreign shocks to output and employment is weak, and the sign of the response of output and employment to Foreign shocks depends on parameter values. The assumption that individuals cannot work in both sectors leads to a lower elasticity of the marginal costs with respect to output.

The remainder of the chapter is organised in the following way. Section 5.2 presents the equations of the model, but with the assumption that individuals can supply their hours of work contemporaneously to both sectors. Section 5.3 presents the findings regarding the transmission of shocks to consumption, real and nominal exchange rates, real output and employment, and Section 5.4 concludes.

5.2 The model without the choice of allocations

In order to investigate how the shocks are transmitted to the macroeconomic variables, another version of the model of the previous chapter, in which individuals can contemporaneously work in both sectors, has been solved. This Section illustrates how to derive the equations for the model in which individuals can work contemporaneously in both sectors, at the same wage. This is a standard assumption in new open economy models with tradeables and nontradeables. In this way, by comparing the two models, with and without the choice of allocations, it is possible to assess whether the restriction that individuals can work only in one sector introduces any change in the dynamics.

All the other assumptions are unchanged, and the functional forms adopted are the same. Therefore, it is not necessary to go through all the equations that describe the model without allocations, but it sufficient to focus only on those that are different. Here follows a synthetic description of the Home country; the Foreign country is parallel and therefore its description is omitted for simplicity.

Individuals do not have probabilities added to their consumptions sets. Instead, the lifetime utility of a Home individual of type i is as follows:

$$U(i) = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(i)) + L\left(\frac{M_t(i)}{P_t}\right) + F(G_{TH,t}, G_{N,t}) + v(\Gamma - \psi - \mathbf{h}_t(i)) \right] ,$$

where $\mathbf{h}$ is an aggregate of hours worked in the two sectors. The share of firms producing tradeables is denoted by n in the Home country and by n^* in the Foreign country. Therefore, Home firms and the goods they produce are indexed by $f_{TH} \in [0, n]$ for the tradeable sector and $f_N \in [n, 1]$ for the nontradeable sector. In the Foreign country, the indexes are $f^*_{TF} \in [0, n^*]$ for the tradeable sector and $f^*_N \in [n^*, 1]$ for the nontradeable sector. Aggrégate hours worked by individual i at date t are thus given by:

$$\mathbf{h}_t(i) = \mathbf{h}_{TH,t}(i) + \mathbf{h}_{N,t}(i) = \int_0^n h_{TH,t}(i, f_{TH})\, df_{TH} + \int_n^1 h_{N,t}(i, f_N)\, df_N \ ,$$

and her budget constraint is given by:

$$\begin{aligned} B_t(i) P_{T,t} + M_t(i) \ \leq \ & (1 + r_{t-1}) B_{t-1}(i) P_{T,t} + M_{t-1}(i) + TR_t(i) + +w_t(i)\mathbf{h}_t(i) + \\ & + \int_0^1 \Pi_{TH,t}(i, f_{TH})\, df_{TH} + \int_0^1 \Pi_{N,t}(i, f_N)\, df_N - P_t C_t(i) \ . \end{aligned}$$

The first-order condition with respect to hours worked becomes:

$$v'(\Gamma - \psi - \mathbf{h}_t(i)) = \frac{u'(C_t(i))}{P_t} w_t(i) \frac{\eta_1 - 1}{\eta_1} \ . \tag{5.1}$$

There are also some modifications at the level of the firms. The aggregators $\widetilde{h}_{TH}(f_{TH})$ and $\widetilde{h}_N(f_N)$ and the demands for individual types of labour are modified as follows:

$$\widetilde{h}_{TH,t}(f_{TH}) = \left(\int_0^1 h_{TH,t}(i, f_{TH})^{\frac{\eta_1 - 1}{\eta_1}} di \right)^{\frac{\eta_1}{\eta_1 - 1}} \ , \quad \widetilde{h}_{N,t}(f_N) = \left(\int_0^1 h_{N,t}(i, f_N)^{\frac{\eta_1 - 1}{\eta_1}} di \right)^{\frac{\eta_1}{\eta_1 - 1}} \ ,$$

$$h_{TH,t}(i, f_{TH}) = \left(\tfrac{w_t(i)}{W_t} \right)^{-\eta_1} \widetilde{h}_{TH,t}(f_{TH}) \ , \quad h_{N,t}(i, f_N) = \left(\tfrac{w_t(i)}{W_t} \right)^{-\eta_1} \widetilde{h}_{N,t}(f_N) \ . \tag{5.2}$$

The production functions in the two sectors are the same, the consumption and price indexes C, C_T, P and P_T stay the same, while the indexes that aggregate individual goods are slightly modified to include the size of the sectors. For example C_{TH} and P_{TH} are given by:

$$C_{TH,t}(i)=\left[\left(\frac{1}{n}\right)^{\frac{1}{\eta_2}}\int_0^n c_{TH,t}(i,f_{TH})^{\frac{\eta_2-1}{\eta_2}}\,df_{TH}\right]^{\frac{\eta_2}{\eta_2-1}},$$

$$P_{TH,t}=\left(\left(\frac{1}{n}\right)\int_0^n p_{TH,t}(f_{TH})^{1-\eta_2}\,df_{TH}\right)^{\frac{1}{1-\eta_2}}.$$

The other aggregate indexes C_{TF}, C_N, G_{TH}, G_N, Y_{TH}, Y_N, P_{TF} and P_N are written following the same logic. Consequently, taking good TH as an example, demand at date t is given by:

$$y_{TH,t}(f_{TH})=\left(\frac{1}{n}\right)\left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t}}\right)^{-\eta_2}\left(C_{TH,t}+C^*_{TH,t}+G_{TH,t}\right)$$

Here I describe briefly the log-linearised equations of the model without the choice of allocations. Since all the other assumptions and the functional forms stay the same, there are only minor changes. The two sectors pay the same wage, therefore the nominal wage that appears in the forward-looking equation for inflation (given in Appendix 4.A) is not sector-specific but common to both sectors. Everything else in the forward-looking equation for inflation is unchanged.

The adjustment in the labour input takes place only along the intensive margin. Individuals' labour supply, obtained by log-linearising the first-order condition (5.1), is for aggregate hours worked in both sectors, while firms' labour demands, obtained by log-linearising equations (5.2) and production, are sector-specific. Therefore, the demand side determines how aggregate hours supplied are allocated across sectors.

Finally, the choice of parameter values in the model without the choice of allocations is as follows. The discount factor β is set to 1/0.04. The steady state terms of trade is determined following the same procedure described in Section 4.3. The goal of the procedure described in Section 4.3, which follows Obstfeld and Rogoff (1995), is to determine the terms of trade endogenously, without making any normalization of the unconditional means of the exogenous productivity processes. Such normalization may be difficult to justify in an empirical application of the model. Utility from leisure is given by $v(x)=\kappa\frac{x^{\omega}}{\omega}$. The preference parameters κ, ψ and ω are determined endogenously so that hours worked in the steady state are equal to 324.8/1369 and the Frisch elasticity of labour supply is in the range of estimates found by MaCurdy (1981). In this way the choice of parameter values is analogous to the one adopted for the model with the choice of allocations.

5.3 The transmission of shocks

In order to see the transmission of shocks in the model, the examination of the impulse responses generated by the numerical solution is complemented by an analytical approach, based on the log-linearised equations. The aim of the analytical approach is to get more insight by combining with each other the approximated, log-linearised equations, thus reducing the complex system to a condensed form.

The first relationship that forms the basis of the analytical approach is an equilibrium condition for the money market, obtained by combining equations (4.47) to (4.53), and using the log-linearised equation (4.39):

$$\left(\widehat{M}_t - \widehat{M}_t^*\right) - \left(\widehat{P}_t - \widehat{P}_t^*\right) = \frac{\sigma}{\varepsilon}\left(\widehat{C}_t - \widehat{C}_t^*\right) - \frac{1}{\varepsilon}\frac{\beta}{1-\beta}\left(E_t\widehat{e}_{t+1} - \widehat{e}_t\right) . \tag{5.3}$$

Equation (5.3) will be referred to as the *money market equilibrium condition.* It is an equilibrium condition which equates (in terms of changes) demand and supply of relative real money balances in both countries, $\left(\widehat{M}_t - \widehat{P}_t\right) - \left(\widehat{M}_t^* - \widehat{P}_t^*\right)$. Demand for real money balances increases with consumption and decreases with the nominal interest rate. Equation (5.3) says that, after an increase in nominal money supply at Home relative to Foreign, the equilibrium in the money market can be restored by either an increase in relative prices, or by an increase in relative consumption, or by an expected appreciation of the Home currency (so that the Home nominal interest rate falls below the Foreign), or by a combination of these.

The second relationship is obtained by subtracting from the log-linearised Home budget constraint (4.47) its Foreign counterpart. Consumption and output changes can be substituted out by using the log-linearised equations (4.33) to (4.36) and their Foreign counterparts, and the log-linearised demand equations for tradeable output. The resulting equilibrium condition is:

$$
\begin{aligned}
dB_t &= \frac{1}{\beta} dB_{t-1} - \frac{1-\beta}{\beta}\frac{1}{k_4}\left(\widehat{C}_t - \widehat{C}_t^*\right) + \frac{1-\beta}{\beta}\frac{1}{k_4}\left(\widehat{P}_t^* + \widehat{e}_t - \widehat{P}_t\right) \\
&\quad + \frac{1-\beta}{\beta}\frac{1}{k_4}\left[\theta\left(1 + k_1 - k_1^*\right) - \phi\left(k_1 - k_1^*\right) - 1\right]\widehat{T}_t \, . \qquad (5.4)
\end{aligned}
$$

Equation (5.4) will be referred to as the *current account equation.* It posits a negative relationship between changes in bond holdings and changes in relative consumption, and a positive relationship between changes in bond holdings and changes in the real exchange rate. The relationship between changes in bond holdings and changes in the terms of trade depends on the parameters of the model. An increase in the (Home) terms of trade makes Foreign tradeable goods more expensive compared to Home tradeable goods, and it affects the relative price of tradeable to nontradeable goods in both countries. The parameters of the model determine the size of the shift in demand from Home to Foreign tradeables (expenditure-switching effect), and the size and direction of the shift in demand from or to nontradeables in both countries.

The third relationship is derived from the log-linearised Euler equations for consumption, by subtracting (4.51) from (4.50):

$$
\sigma E_t\left(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*\right) - \sigma\left(\widehat{C}_t - \widehat{C}_t^*\right) = E_t\left(\widehat{P}_{t+1}^* + \widehat{e}_{t+1} - \widehat{P}_{t+1}\right) - \left(\widehat{P}_t^* + \widehat{e}_t - \widehat{P}_t\right) \, . \quad (5.5)
$$

Because it describes the optimal intertemporal allocation of changes in relative consumption $\frac{C}{C^*}$, I can call equation (5.5) the *Euler equation for relative consumption* in both countries. It posits a positive relationship between the expected growth rate in relative consumption and the expected growth rate of the real exchange rate. The real exchange rate is defined as $\frac{eP^*}{P}$, thus an increase (fall) in this quantity is a "depreciation" ("appreciation").

Equations (5.3), (5.4) and (5.5) constitute a dynamic system of three equations in five unknowns, where the unknowns are changes in relative consumption, $\widehat{C}_t - \widehat{C}_t^*$, in the real exchange rate $\widehat{P}_t^* + \widehat{e}_t - \widehat{P}_t$, in the nominal exchange rate $\widehat{e}_t$, in bond holdings dB_t, and in the terms of trade, $\widehat{T}_t$. Even if the number of equations is lower than the number of unknowns, equations (5.3) to (5.5) still have a lot to say about the impact of shocks in the model.

In particular, consider the baseline parametrization. The parameters ϕ and θ are equal to one, δ is equal to δ^*, and this guarantees that k_1 and k_1^* are equal. As a result, $\widehat{T}_t$ disappears from equation (5.4). Moreover, since σ is equal to one, equation (5.5) implies that the expected growth rate of relative consumption is always equal to the expected growth rate of the real exchange rate. Using this last fact and recalling that $\frac{1}{\beta}$ is greater than one, it is possible to conclude from equation (5.4) that dB_t is stable as t goes to infinity if and only if:

$$\widehat{C}_t - \widehat{C}_t^* = \widehat{P}_t^* + \widehat{e}_t - \widehat{P}_t \, . \tag{5.6}$$

Given that the solution method explicitly looks for the path that does not violate stability[5], in the baseline parametrization relative consumption changes are always equal to real exchange rate changes. Shocks have an impact on the real exchange rate only if they affect relative consumption, and vice versa. Intuitively, the above relationship would not hold with equality in some departures from the baseline parametrization, but the model would always retain a strong link between relative consumption and real exchange rate changes.

Incidentally, equation (5.6) characterizes a complete markets allocation. Therefore it can be inferred from the above discussion that the model reproduces a complete markets allocation in any parametrization such that: i) $\widehat{T}_t$ disappears from equation (5.4), and ii) σ is equal to one.

The last equation can be used to solve forward equation (5.3) in the baseline parametrization with $\varepsilon = 1$, obtaining:

$$\widehat{e}_t = (1-\beta)\, E_t \sum\nolimits_{j=0}^{\infty} \beta^j \left(\widehat{M}_{t+j} - \widehat{M}_{t+j}^*\right) \tag{5.7}$$

The above relationship says that the nominal exchange rate depends only on current and future expected changes in relative money supply, and is unaffected by the other exogenous shocks in the model[6]. The nominal exchange rate may be affected by other

[5]Individuals observe the shocks at date t and form expectations conditional upon that realisation of the shocks. Given that their optimal allocation of changes in relative consumption is dictated by the Euler equation for relative consumption, the only way for them to prevent in expectation an explosive path for dB_t in the log-linearised solution is by setting $\widehat{C}_t - \widehat{C}_t^* = \widehat{P}_t^* + \widehat{e}_t - \widehat{P}_t$.

[6]For example, if a shock of size ϵ in the Home nominal growth rate of money occurs at date t, and no other Home or Foreign shocks occur afterwards, then the response of the nominal exchange rate is $\widehat{e}_t = \frac{\epsilon}{1-\beta\rho}$, where ρ is the autoregressive parameters for the Home nominal money growth rate.

shocks in some other parametrizations, but intuitively the model always retains a strong link between money and the nominal exchange rate. Fluctuations in money supply are the exclusive or the main cause of fluctuations in the nominal exchange rate.

In conclusion, equations (5.3) to (5.5) can be used to illustrate that in the baseline parametrization there is a perfect correlation between changes in relative consumption and changes in the real exchange rate, and between relative money supply and the nominal exchange rate. For this reasons, equations (5.3) to (5.5) can also be used to understand how departures from the baseline parametrization affect these correlations for different types of shocks.

The effects of aggregate shocks can be obtained simply as the sum of sectoral shocks.

The response of consumption

Figure 12 shows the response of consumption in the baseline parametrization to one percent shocks in the exogenous processes. The two monetary shocks have the highest impact in the short run, productivity shocks have a much smaller impact, but they are quite persistent, and government expenditure shocks have no impact on consumption. Moreover, sensitivity analysis has shown that the order of importance between monetary and productivity shocks is a stable feature of the model, since it is preserved under different parameter values. Therefore, according to the model, if shocks all have the same variance (and the same quarterly frequency), then the main source of short-run fluctuations in consumption are monetary shocks, while productivity shocks are responsible for medium to long-run fluctuations.

Let us analyse the transmission of monetary shocks first. An increase in nominal money supply at Home causes a depreciation of the Home currency, an increase of the Home consumer price index, and a fall in the real interest rate. As a result of the lower real interest rate, Home and Foreign consumption increase. The fact that increases in nominal money supply at Home have a positive impact on Foreign consumption can be inferred from Figure 12, by looking at the positive impact of Foreign money supply on Home consumption. Due to the symmetry of the baseline parametrization, responses to Foreign shocks are the same as the responses of Foreign variables to Home shocks.

Then, the transmission of productivity shocks. Positive shocks in productivity trigger a fall in prices, and thus an increase in real money balances and consumption. Since shocks in Foreign nontradeables do not affect Home prices, they are not transmitted to

Home consumption. Figure 12 shows that, in the baseline parametrization, shocks in $\widehat{z}_N$ have more impact on Home consumption than shocks in $\widehat{z}_{TH}$ or $\widehat{z}^*_{TF}$. This happens because, in this parametrization, nontradeable goods are given the same weight as the composite bundle of Home and Foreign tradeables, and as a result Home nontradeables productivity shocks have more impact on the Home CPI than either kind of tradeable productivity shock. Because there is no physical capital, the persistence of the effects of productivity shocks is a very interesting feature of the model. Productivity shocks are transmitted through the fall in prices, but this can happen only gradually, because of price rigidities. As a result, productivity shocks have persistent effects on real money balances and consumption. The persistence of the effects of productivity shocks is due to the nominal rigidities and does not require the presence of capital in the model.

Finally, the transmission of government expenditure shocks. Positive government expenditure shocks increase output, thus they have both a positive and negative effect on the left-hand side of equation (4.47) in Appendix 4.B. However, the net effect of government expenditure on the current account is zero, as equation (5.4) shows. Equation (5.4) is obtained by substituting out of the Home and Foreign aggregate resource constraints a market clearing condition for Home and Foreign tradeable output. Hence, the increase in output demand induced by a positive government expenditure shock is exactly counterbalanced by an analogous increase in output supply, with no effects on the current account and consumption. Sensitivity analysis has shown that the neutrality for consumption of government expenditure shocks is a stable feature of the model, with the exception of decreasing marginal labour productivity. Chart 1 shows that if $\alpha < 1$ government expenditure shocks have a negative impact on consumption. This happens because increases in government expenditures boost the demand for output; if $\alpha < 1$, firms expand production but increase their prices at the same time. The fact that government expenditure shocks are followed by increases in prices if $\alpha < 1$ can be inferred from equations (4.54) to (4.57). The increases in prices explains why government expenditure shocks have a negative impact on consumption if $\alpha < 1$.

By comparing the responses in Figure 12 to the responses in the model without the choice of allocations (Figure 13), it is possible to assess how the dynamics of consumption is affected by the introduction of probabilities in the utility function. The different responses in Figures 12 and 13 are due to the slope of the labour supply, and the response of real wages, in the two cases. In the model without the choice of allocations the sectoral, log-linearised labour supply has a "standard" slope, that is, increases in hours worked require an increase in the real wage. However, in the model with the choice of allocations, changes in labour supply (in each sectoral labour market) are not associated with changes in real wages[7]. Therefore, while in the model with the choice of allocations output increases do not entail increases in the real wages, in the model without the choice of allocations shocks that cause output to increase are followed by increases in real wages, which in turn have a positive effect on prices, because of the firms' pricing behaviour.

For example, let us consider a positive Home or Foreign monetary shock. In the model without the choice of allocations this is followed by an increase in real wages, consequently, there is a higher increase in prices and a lower impact on consumption, compared to the model where individuals can choose the probabilities of working in sectors. Next, let us consider a positive Home or Foreign productivity shock. This causes a decrease in Home or Foreign wages. In the model without the choice of allocations the fall in wages is more pronounced (because it is tied in with a fall in hours), and as a result, the consequent fall in prices and the positive impact on consumption are more pronounced. Lastly, consider a positive Home or Foreign government expenditure shock. Its effect is to increase Home or Foreign output. In the model without the choice of allocations real wages go up with output, therefore prices increase too, and by this way the effect on consumption is negative.

Figures 14 and 15 show the response at impact of consumption after a Home monetary shock, for different values of σ, ϕ and θ. Changes in these parameters are associated with permanent wealth effects, which influence consumption. For example, if $\phi > \frac{1}{\sigma}$ individuals are more willing to postpone the consumption of tradeable goods and lend units of C_T abroad: a positive Home monetary shock induces a positive wealth effect in the Home country, and the response of Home consumption becomes larger. If $\theta > 1$, TH and TF goods are close substitutes and a positive Home monetary shock induces a positive effect on the trade balance of the Home country, thus the response of Home consumption becomes larger.

[7] This result follows from the log-linearized first-order conditions for labour supply in the model with the choice of allocations, and from the fact that $\widehat{\mathbf{h}}_{TH,t} = \widehat{\mathbf{h}}_{N,t} = 0$.

The economic intuition of this result is the following. Because the disutility of work is convex in hours (but linear in probabilities), individuals want to keep working hours always constant and equal to the steady state values. Therefore, all the adjustment takes place along the extensive margin. Individuals only accept the change in the real wage that will keep their hours constant, and the only way for them to achieve this result is to accommodate all the labour demand from firms. In each of the two sectoral labour markets, the log-linearized labour supply (in probabilities) is infinitely elastic.

The response of the real exchange rate

Figure 16 shows the response of the real exchange rate in the baseline parametrization to one percent shocks in the exogenous processes. Because of the above-mentioned link with relative consumption changes, the above-mentioned symmetry and the fact that shock have the same size, the response of the real exchange rate to a shock is the same as the responses of Home consumption to the same shock, minus the response to that kind of shock occurring in the other country[8].

Monetary shocks have the highest impact in the short run, productivity shocks in nontradeables have a smaller impact in the short run but are quite persistent, while productivity shocks in tradeables and government expenditure shocks have no impact in the baseline parametrization. The model thus explains the short-run fluctuations in the real exchange rate as due to monetary shocks, while productivity shocks in nontradeables are responsible for medium to long-run fluctuations, assuming equal variance for all shocks.

Let us analyse the transmission of monetary shocks first. An increase in money supply at Home causes a nominal depreciation of the Home currency, an increase in the Home consumer price index, and a fall in the Foreign consumer price index. Because of price rigidity, the effect on the nominal exchange rate prevails, therefore a positive Home money supply shocks brings about a depreciation (increase) in the real exchange rate. By the opposite mechanism, a positive Foreign money supply shock causes an appreciation (decrease) in the real exchange rate.

Then, the transmission of productivity shocks. Positive productivity shocks in nontradeables Home (Foreign) goods cause a decrease in Home (Foreign) prices, thus they cause an increase (decrease) in the real exchange rate. In the baseline parametrization, shocks in tradeable goods have the same impact on both the Home and Foreign CPI, therefore their impact on the real exchange rate is zero.

Finally, the transmission of government expenditure shocks. As explained above, government expenditure shocks are neutral for consumption, except when α is less than one. Only in this case then government expenditure shocks have an impact on the real exchange rate. In all the other cases, since relative consumption does not change, there are no effects on the real exchange rate, as equation (5.5) makes clear.

[8] For example, the response of the real exchange rate to a one percent shock in μ is the same as the response of Home consumption to the one percent shock in μ minus the response to a one percent shock in μ^*.

Differences in the responses of the model without the choice of allocations are caused by the behaviour of real wages and prices, explained above (see Figure 17).

The neutrality of productivity shocks in tradeables shows up only under the symmetric, "baseline" parametrization. Productivity shocks in tradeables cause a fall in both the Home and the Foreign price levels. In the baseline parametrization these falls are identical, therefore the effect on the real exchange rate is zero. However, if the proportions in consumption of tradeable goods of the same type are different in the two countries, then the response of the real exchange rate is not equal to zero, and becomes indeterminate in sign, as Figure 18 illustrates. Therefore, Figure 18 shows that the model's implications for the fluctuations in the real exchange rate may be biased if δ and δ^* are not estimated correctly.

The response of the real exchange rate to monetary and productivity shocks may also be affected by the elasticity of substitution between Home and Foreign tradeable goods. Figures 19 and 21 report the responses at impact of the real exchange rate obtained by varying simultaneously θ and δ, while δ^* is changed specularly. If $\delta = 1 - \delta^*$ is less than 0.5, there is *home bias* in the model[9]. When $\delta = 1 - \delta^*$ is low, the fall in Home prices after a positive productivity shock is higher than the fall in Foreign prices, and as a result the real exchange rate goes up. The elasticity of substitution θ determines the size of the switch in demand from TF to TH goods. If θ is higher than one, more consumers substitute Foreign tradeables with Home tradeables, thus causing a nominal appreciation so as to bring the goods market back into equilibrium. On the other hand, if θ is lower than one, less consumers substitute Foreign tradeables with Home tradeables, and as a result the nominal exchange rate depreciates, strengthening the positive effect on the real exchange rate in the home bias situations. In conclusion, the message of Figure 21 for the Balassa-Samuelson hypothesis is the following. A positive productivity shock in the tradeable goods sector may cause an increase in the real exchange rate, but this effect is very small unless there is a high degree of home bias and θ approaches zero (a situation of almost no substitutability between TH and TF goods).

Departures from $\theta = 1$ are associated with the emergence of permanent wealth effects,

[9] Warnock (2003) defines home bias as a situation where at any given relative price, Home consumers consume more home produced tradeable (relative to foreign produced tradeable) than do Foreign consumers. Since:
$\frac{C_{TH}}{C_{TF}} = \frac{1-\delta}{\delta}\left(\frac{P_{TH}}{P_{TF}}\right)^{-\theta}$ and $\frac{C^*_{TH}}{C^*_{TF}} = \frac{1-\delta^*}{\delta^*}\left(\frac{P^*_{TH}}{P^*_{TF}}\right)^{-\theta}$,
then home bias arises in the model whenever $\delta < \delta^*$.

as shown in Figures 20 and 22. However, these wealth effects influence only marginally the response of the real exchange rate to monetary shocks, and are more relevant in the case of productivity shocks.

The response of the nominal exchange rate

Figure 23 shows the response of the nominal exchange rate, in the baseline parametrization, to one percent shocks in the exogenous processes. As explained in Section 5.3, in this parametrization the nominal exchange rate is only affected by monetary shocks. Moreover, monetary shocks, as it can be deduced from equation (5.7), are symmetric: if a nominal, positive shock of size ϵ occurs at date t, and no other shock occurs afterwards, then the response of the nominal exchange rate is $\widehat{e}_t = \frac{\epsilon}{1-\beta\rho}$ for a shock in the Home nominal growth rate of money, and $\widehat{e}_t = -\frac{\epsilon}{1-\beta\rho}$ for a shock in the Foreign nominal growth rate of money, assuming a common autoregressive parameter ρ. The same is true in the baseline parametrization of the model without the choice of allocations, for this reason the impulse responses of the nominal exchange rate are not shown for that model, as they are identical.

Departures from the baseline may entail different responses, but, intuitively, the model will always retain a strong link between nominal shocks and the nominal exchange rate. The formulas given above show that the parameters β and ρ govern the response of the nominal exchange rate to monetary shocks; changes in other parameters may generate some effects, albeit quantitatively small.

For example, Hau (2000) proves that nontradeables have an "exchange rate magnification effect", that is, a high proportion of nontradeables increases the volatility of the nominal exchange rate. Quantitatively, this effect is quite small[10], but it is one of the established facts of the new open economy literature. However, the sensitivity analysis shown in Figures 24 to 27 suggests that parameters other than γ affect more the response of the nominal exchange rate to monetary shocks.

The elasticity of substitution between tradeables and nontradeables ϕ affects the response of the nominal exchange rate in both models, with and without the choice of allocations. Higher values for ϕ are associated with lower responses of the nominal exchange rate after a positive Home monetary shock. This happens for the following reason.

[10] As shown by Hau on page 441, Figure 2. More nontradeables reduce the impact of monetary shocks on the CPI, therefore, the nominal exchange rate moves more in order to bring the money market back to equilibrium.

A positive Home monetary shock causes a nominal depreciation, an increase of the Home tradeable goods price index P_T, and a decrease of the Foreign tradeable goods price index P_T^*. The nominal depreciation makes the TF goods more expensive for Home consumers, and the TH goods less expensive for Foreign consumers. The higher is ϕ, the higher will be the substitution towards nontraded goods and the fall in demand for TF goods from Home consumers. Similarly, the higher is ϕ, the higher will be the substitution away from nontraded goods and the increase in demand for TH goods from Foreign consumers. As a result, for higher values of ϕ a lower increase in e ensures that the demand for TF (TH) goods does not fall (increase) too much, and the goods market can stay in equilibrium.

Moreover, it is not always true that more nontradeables increase the response of the nominal exchange rate to monetary shocks. In fact, in the model without the choice of allocations (Figure 25), if ϕ is less than one more nontradeables increase the response of e, but the opposite happens if ϕ is higher than one. This can be explained by recalling that in the model without the choice of allocations changes in output are associated with changes in real wages. Simulation results have shown that, in the model without the choice of allocations, if ϕ is greater than one Foreign real wages fall after a Home monetary expansion, and this fall is less pronounced if there are less nontradeables (γ low). The fall in Foreign real wages induces Foreign firms to reduce their prices, contributing towards the increase in Foreign real balances. Since the latter effect is less pronounced if there are less nontradeables, it is possible to understand why whenever $\phi > 1$ the nominal exchange rate increases more if γ is low. On the other hand, simulation results show that if ϕ is less than one Foreign real wages increase after a Home monetary expansion, and the increase is less pronounced if there are more nontradeables (γ high). The increase in Foreign real wages induces Foreign firms to increase their prices, dampening demand. Therefore, in the model without the choice of allocations, whenever $\phi < 1$ the nominal exchange rate increases more if γ is high.

The elasticity of substitution θ and $\delta = 1 - \delta^*$ affects the response of the nominal exchange rate to monetary shocks in both models (Figures 26 and 27). A nominal depreciation makes TH goods more attractive, however, for the same Home positive money shock, the nominal depreciation must be lower for higher levels of θ, since the higher is θ, the higher is the switch in demand from TF to TH goods. This explains why the nominal exchange rate is very volatile for low values of θ. The parameters δ and δ^* determine the size of the expenditure-switching effect, therefore they govern also how much e can change while keeping the equilibrium in the goods market.

The response of output and employment

As it can be seen from Figure 28, the response of Home real output[11] to Home monetary shocks is the highest in the short run, while productivity shocks in the Home country and Home government expenditure shocks have a higher response in the long run. Then, if all shocks have the same variance, the model identifies monetary shocks as the main source of short-run output fluctuations, and productivity and government expenditure shocks as the main source of medium to long-run output fluctuations.

Let us analyse the transmission of monetary shocks first. An increase in Home money supply has a positive effect on Home and Foreign consumption and causes a depreciation of the Home currency. Home output is driven up partly by the increase in Home and Foreign consumption, and partly by the depreciation, which makes Home tradeable goods cheaper. However, Foreign money shocks have no effect on Home output. This happens because of the values chosen for the elasticities of substitution in the baseline parametrization. After a positive Foreign monetary shock, the Home currency appreciates and Home tradeable goods become more expensive compared to Foreign tradeable goods. Under the baseline parametrization, after a positive Foreign money shock demand increases only for Foreign goods, while the demand for Home tradeables and nontradeables is unaffected.

Then, the transmission of productivity shocks. It is interesting to note that, without capital accumulation, productivity shocks produce a hump-shaped response that peaks at roughly one quarter. The persistence of the response to productivity shocks is explained by nominal rigidities. After a positive productivity shock in the Home country, Home firms reduce their prices (because their marginal cost is lower), thus inducing individuals to increase consumption. Output increases to meet the demand for consumption, however, because of price rigidity the fall in prices is gradual, so the impact on consumption and output is long-lasting. Finally, government expenditure shocks increase the demand for output and are never transmitted abroad, as governments purchase only tradeable and nontradeable goods produced in their own country.

The responses of output to the shocks in the model without the choice of allocations are qualitatively similar, as shown by Figure 29. However, their sizes are markedly different,

[11] Here real output is the sum of tradeable and nontradeable output, evaluated at constant, steady state, prices.

since the responses to money and government shocks are dampened, and the responses to productivity shocks are amplified, compared to the model with the choice of allocations. This happens because increases in output entail increases in hours worked, but in the model without the choice of allocations increases in hours drive real wages up. As a result, firms pass part of the increase in real wages onto prices, curbing demand. The explanation of why the response of output to productivity shocks is amplified in the model without the choice of allocations also revolves around labour supply. Contrary to monetary and government expenditure shocks, positive productivity shocks are associated with reductions in labour demand[12]. As a result, hours worked fall, and in the model without the allocations choice real wages must fall until labour supply becomes equal to demand. Then, the fall in real wages results into lower prices and higher output demand. Therefore, in the model without the choice of allocations, productivity shocks are followed by a marked increase in output, which is relatively stronger than in the model with the choice of allocations.

Figures 30 to 33 illustrate that if the assumptions of unitary elasticities ϕ and θ are relaxed, then Foreign monetary and nontradeable productivity shocks have an impact on Home output, but they generate only very small effects that are indeterminate in sign. In fact, if the elasticities ϕ and θ are less than one, then Foreign shocks have a positive impact on output, otherwise the sign is reversed. After a positive Foreign monetary shock, consumption increases in both countries, and the Home currency appreciates. Because of the nominal appreciation, P_T goes down but P_T^* goes up, hence the aggregate basket of tradeable goods becomes relatively less expensive (compared to nontradeables) in the Home country, and relatively more expensive in the Foreign country. Moreover, Home tradeable goods become relatively more expensive compared to Foreign tradeable goods. When either ϕ or θ are low, individuals in the Foreign country (where the response of consumption is higher) substitute little towards, respectively, nontradeables or Foreign-produced tradeables. However, when either ϕ or θ are high, substitution effects are more pronounced, and a positive Foreign monetary shock has a negative impact on Home tradeable goods and Home output. In the case of Foreign productivity shocks, their main consequence is the fall of prices in the relevant sectors and the increase in Foreign consumption. If ϕ or

[12]Positive monetary and government expenditure shocks are associated with increases in hours worked (because output increases drive up hours), while positive technology shocks cause a decrease in hours worked (because firms require less labour).

θ are low, substitution effects are small, and the increased consumption demand in the Foreign country results in more demand for Home tradeable goods and output.

Figure 34 shows the impulse responses of total Home employment, which is defined as the sum of the employment rates in the two sectors. Using equations (4.1) and (4.2), it can be shown that the responses of total employment are the same as the responses of real output presented in Figure 28, except for productivity shocks and in the case of decreasing labour productivity. For this reason, Figure 34 shows the responses with decreasing labour productivity alongside the responses with constant labour productivity.

Total employment increases after a positive monetary or government expenditure shock, since firms can satisfy the higher output demand only by increasing the labour input. In the case of decreasing labour productivity, firms must employ more workers in order to satisfy demand. On the other hand, after a positive productivity shock more output can be produced with less labour, therefore employment falls. With decreasing labour productivity firms cut more workers because labour is less productive, since one worker produces less than one unit of output (adjusted for productivity). Moreover, sensitivity analysis shows that if the assumptions of unitary elasticities ϕ and θ are relaxed, then Foreign monetary and nontradeable productivity shocks can have an impact on Home employment, as one could expect by looking at Figures 30 to 33. These effects are positive for low values of ϕ and θ, and negative for values of ϕ and θ greater than one, both for productivity and monetary shocks.

5.4 Conclusion

The move towards an intertemporal general equilibrium model with explicit microfoundations constitutes a rather recent "paradigm shift" for the study of the international transmission of shocks. However, the transmission of shocks in the *Redux* or in its extensions is different, both quantitatively and qualitatively, from the traditional Mundell-Fleming-Dornbusch model. The model presented in this chapter uses very general functional forms, to ensure comparability with similar models, and the analysis of the transmission of shocks is conducted also for the case in which individuals can work in both sectors.

The insights that can be gained from the sort of theoretical analysis presented in this chapter may thus be useful to all those researchers looking for a better model to match the data. In fact, since the chapter explains the effects of changes in parameter values on the transmission mechanism, it is possible to evaluate whether the empirical calibration of this model or a similar one is likely to be successful, or else some change in the assumptions is needed. This point can be illustrated with reference to the main findings of the chapter.

The strong impact of money on all variables in the short run may be altered by resorting to a different calibration of the shock[13], but the absence of persistence is an inherent feature of this and similar models, since changes in parameters did not have a noticeable effect on it.

The effects of government expenditure shocks on output have been shown to be sensitive to changes in parameters: this means that there are some gains that can be exploited by some suitable empirical calibration and testing procedure. However, under all parametrizations the responses of output to government expenditure shocks were rather weak and poorly persistent, calling for a change in some of the modelling assumptions. A change is also needed to allow government expenditure shocks having an effect on the real exchange rate. But on the positive side, the assumption that individuals cannot work in both sectors seems to be a desirable feature to have in the model, since it prevents government expenditure shocks having a negative effect on consumption.

Lastly, the effects of Foreign shocks on Home output and employment. The message that can be inferred from Figures 30 to 33 is twofold. On the one hand, the weak transmission of Foreign shocks to Home output calls for a change in the modelling assumptions [14]. For example, some degree of pricing to market would reduce the expenditure switching effect, and generate a positive co-movement of output. On the other hand, since changes in the elasticities of substitution generate responses having different size and magnitude, the model is very flexible[15]. In particular, by allowing the elasticities to differ across countries, it could be possible to reproduce the internationally asymmetric effects of money shocks[16]. Since there are no papers that estimate two-country models by looking at the responses

[13] For example, the shock may be in the level instead of in the growth rate of money. In this Chapter monetary policy is not specified by means of an interest rate rule. This is because shocks to the interest rate rule are not unambiguously monetary in nature, furthermore, with an interest rate rule it is not possible to isolate the genuine transmission mechanism from the monetary policy reaction, since the response of all variables depends on the behaviour of the central bank.

[14] Betts and Devereux (2001), and Kim (1999) find a strong positive response of foreign outputs to US expansionary monetary shocks.

[15] In the Mundell-Fleming-Dornbusch model, under flexible exchange rates a monetary expansion in one country produces a negative output response in the other country. See Borondo (2002).

[16] The empirical literature has highlighted that shocks originating in the US seem to have different effects on European variables than shocks originating in Europe on US variables. See, for example, Artis, Osborn and Perez (2004).

to foreign shocks, I think that this very interesting area of research is still unexplored.

Conclusions

The thesis applies a variety of DSGE models to a set of problems, all of them related with the study of economic fluctuations. Their summarised conclusions constitute the point of departure for an appraisal of the DSGE methodology, from the particular point of view of the analysis of fluctuations.

Chapter 2 documents the crucial features of economic fluctuations in Italy, namely the high volatility of hours worked and the low volatility of employment, and explains why the standard RBC model cannot reproduce them. In fact, the volatility of hours can be increased by assuming a high elasticity of intertemporal substitution of leisure, or by introducing the possibility of borrowing and lending from the rest of the world. However, results from the simulations have shown that the increase in the volatility of hours always happens at the expense of the volatility of consumption, which becomes too low.

Chapter 2 then concludes that the inability of the standard RBC model in generating a high volatility of hours is due to the nature of the transmission of the technology shock in the RBC model, and the way the persistence is generated. In the RBC model the motive for changing the labour supply is intertemporal substitution, but agents want to spread the payoff of working longer hours into the future, via capital accumulation. As a result, when the responsiveness of hours of work is increased (via a change in the labour supply elasticity), the RBC model generates too much consumption smoothing or too high volatility of investment to be consistent with the data.

However, the standard RBC model was applied initially only to the US, therefore it is possible that its original design was heavily influenced by the institutional characteristics of the US economy. When applied to other countries, a suitable adaptation to the specific institutional features may increase the model's ability to match the data. Therefore, the inability of the standard RBC model to reproduce the volatility of hours in Italy may not be inherently due to its propagation mechanism, but rather to the neglect of some specific labour market institutions or features of the Italian economy. In particular, the Italian economy is characterised by a high degree of labour market rigidity, and by the existence of a large underground economy. The presence of labour market rigidities and the sizeable underground sector can explain why in Italy hours of work fluctuate more than employment. Rigidities prevent or make it costlier for registered, "official" firms to change the number of employed workers, and as a result they prefer to change hours instead of employees in response to shocks. Individuals can offset the change in the registered or "official" hours by shifting from the production of goods in the registered sector to the production of goods in the unregistered sector, or vice versa.

Therefore, Chapter 3 modifies the RBC model by adding labour market rigidities and the underground sector, and explains why these modifications are successful in generating a high volatility of hours and a low volatility of employment. However, the modified RBC model is still not completely able to reproduce the fluctuations in the Italian economy, since the standard deviation of consumption is again lower than in the actual data. Nevertheless, the exercise conducted in Chapter 3 is useful because it suggest that, in order to match all the standard deviations in the Italian data, the modified RBC model must be supplemented by some additional assumptions. In fact, as highlighted on page 75, although the model introduced in Chapter 3 captures important institutional differences between Italy and the US (labour market rigidities and the underground sector), it does not, of course, capture all of them; furthermore, the distinctive features of business cycle fluctuations in Italy may also be due to individual preferences and tastes. Thus there are several promising avenues for future research.

Using a two-country model with nominal rigidities, Chapters 4 and 5 illustrate the high potential of DSGE methods in understanding the transmission of shocks, both to relative prices and quantities and between countries. The model that is put forward in Chapters 4 and 5 belongs to the "new open economy macroeconomics" literature initiated by Obstfeld and Rogoff (1995), but the analysis is more comprehensive than the previous studies. This is because: 1) many of the critical parameters already studied in isolation are present, so their role can be analysed simultaneously; 2) the analysis is not confined to monetary shocks; 3) shocks are disaggregated by sector; 4) some novel assumptions are introduced (the assumptions that the marginal productivity of labour may be decreasing, and that individuals cannot work in both sectors). These latter modelling innovations are introduced in a way that preserves comparability with the other papers: constant marginal productivity of labour is treated as a special case, and results are also shown for the case in which individuals can work in both sectors.

In particular, Chapter 4 investigates how shocks are transmitted to the relative price of domestic tradeables versus nontradeables, and to the ratios of domestic tradeable versus nontradeable output and employment. Using a two-country general equilibrium model with monopolistic competition and sticky prices, the chapter shows that not only sector-specific supply or demand shocks affect these relative prices and allocations, but also aggregate monetary shocks, thus contributing to explain why money has sectoral effects, as found by the empirical literature. The effects of all shocks (money, supply and demand) depend on the choice of parameter values.

Chapter 5 investigates the domestic and international transmission of shocks to the main macroeconomic variables. While monetary shocks dominate the responses of all variables but are poorly persistent, demand (government expenditure) shocks have little impact on the main variables. Even without capital, technology shocks are quite persistent. The chapter shows that the sign of the response of output and employment to Foreign shocks is not determined but it depends on parameter values, and the propagation of Foreign shocks to output and employment is weak. The latter result is important because, although the empirical research is still under way in this area, what is known now about the transmission of shocks across countries seems not always to be fitting into the picture coming from the theoretical model. In fact, Betts and Devereux (2001), and Kim (1999) find a strong positive response of foreign outputs to US expansionary monetary shocks. But on the positive side, the theoretical research itself is still under construction, and as the "new open economy" literature keeps expanding, the days in which international macroeconomists will have a framework universally accepted and fully consistent with the data may get closer.

Thus, the experience gained by applying the DSGE methodology to a set of somehow similar problems points to a mixture of successes and failures. In fact, it was possible to reproduce to some extent the facts that characterise the economic fluctuations in Italy, but it was not possible to obtain a perfect match of the standard deviations; it was possible to obtain meaningful responses to exogenous shocks, but the sign and the persistence of some responses were not in line with the evidence. It seems that, decades after the Lucas' critique, macroeconomists are finally equipped with a rigorous framework for policy analysis, but the need to establish all policy recommendations on models that are fully consistent with the empirical evidence is still there. Due to the stylized nature of their early models, Kydland and Prescott (1982), and Long and Plosser (1983), set out to reproduce only second-order moments, but since then the state-of-the-art DSGE modelling has become far more sophisticated, so it seems right to ask for more.

However, the message that can be inferred from the thesis' chapters[17] is that the remaining shortcomings of DSGE modelling do not constitute a lethal threat, capable of putting into question the structure of the models. Indeed, since the DSGE methodology is flexible and capable of incorporating a variety of assumptions, there is always the possibility of finding the "right sort of imperfection" or modelling trick (even adding biases and time-inconsistencies in agents' behaviour), that could reconcile the real world facts with the theory.

Considering the more or less recent advances in DSGE modelling, it is fair to say that this avenue has now been travelled to a great extent. In fact, to give just some examples, some models[18] explain the real effects of money by relying on the assumption that not all the agents optimally adjust their wages or prices. Other models introduce frictions[19] or institutional features that prevent the immediate response of the agents and thus increase the persistence of the effects of the shocks. However, judging in the light of the literature review presented in the previous Section, this approach or avenue is likely to lead away from the desire for a "scientific" or intellectually rigorous discipline, a desire that has been crucial for the construction of the DSGE methodology.

To the remark that employment fluctuations are not all voluntary, because social conventions and institutional structures affect them, Lucas replied that conventions and institutions do not impose themselves on individuals, but instead they are chosen themselves, in order to aid in matching preferences and opportunities[20]. But as more frictions, institutional features or *ad hoc* assumptions are added to the models, the DSGE approach gradually seems to move towards the stage when it will be no more "scientific" than starting directly with an aggregative model. Nonetheless, the strategies that have been tried in DSGE modelling to narrow the gap between the data and the theory have now generated a better knowledge, and, since the discipline of economics will always be evolving, a paradigm must be "tried and tested" first, and perhaps also stretched in all directions,

[17]See, for example, the remarks on pages 75 and 157.
[18]As in Chapters 4 and 5.
[19]As in Chapter 3.
[20]See Lucas (1981), page 4.

before something new can come along.

In conclusion, since the thesis did not aim to be (and could not be) a complete evaluation of the DSGE methodology, based on all problems that can be solved with it, the message that it puts forward is only provisional. Future developments in DSGE modelling will certainly affect profoundly this methodology.

It has been noted in several places in the thesis that DSGE models are successful in reproducing some empirical facts or tendencies, or in generating a better understanding of the transmission of shocks, but there is still ahead some work to do in order to make them fully consistent with the empirical evidence[21]. This work may require modifying individuals' preferences, introducing particular institutions or imperfections in the models, or enlarging the set of shocks, and in doing so it is possible that some of the rigour that characterised the early DSGE models might be lost. However, the introduction of *ad hoc* modifications to improve the match with the data can be seen simply as an intermediate step, moreover, these modifications by themselves can generate a better knowledge of the DSGE methodology. By repeated adjustments, it will be possible to discern one day whether the task of explaining the data can be accomplisher by either some modifications of existing theories, such as adding imperfections, or by some modifications of the key features of the DSGE methodology itself. A lot of work will continue to be done in macroeconomics, and if the empirical and the theoretical literature do not lose sight of each other, its fruits will be long-lasting.

[21]As highlighted on pages 75 and 157.

Bibliography

AIYAGARI, S. Rao (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving", *The Quarterly Journal of Economics*, **109** (3), 659-84.

ALPANDA, Sami, and UEBERFELDT, Alexander (2004), "The Workweek Length in the Real Business Cycle Literature" (mimeo University of Minnesota).

ANDRES, Javier, LOPEZ-SALIDO, J. David, and VALLES, Javier (2002), "Intertemporal Substitution and the Liquidity Effect in a Sticky Price Model", *European Economic Review*, **46**, 1399-1421.

ARTIS, Michael, OSBORN, Denise R., and PEREZ, Pedro J. (2004), "The International Business Cycle in a Changing World: Volatility and the Propagation of Shocks in the G7" (Discussion Paper 4652, CEPR).

BACKUS, David (1999), "EconomicDynamics Interview: David Backus on international business cycles" (*Economic Dynamics Newsletter*,
http://www.economicdynamics.org/News1.htm).

BACKUS, David K., KEHOE, Patrick J., and KYDLAND, Finn E. (1995), "International Business Cycles: Theory and Evidence", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research* (Princeton: Princeton University Press).

BARRO, Robert J., and SALA-I-MARTIN, Xavier (1995), *Economic Growth*, first ed. (New York: McGraw-Hill).

BARTH, Marvin J. III, and RAMEY, Valerie A. (2001), "The cost channel of monetary transmission", *NBER Macroeconomic Annual* (Cambridge, Mass.: MIT Press).

BAXTER Marianne, and Mario CRUCINI (1993), "Explaining Savings - Investment Correlations", *American Economic Review* **83**, 416-436.

BENHABIB, Jess, ROGERSON, Richard, and WRIGHT, Randall (1991), "Homework in Macroeconomics: Household Production and Aggregate Fluctuations", *Journal of Political Economy*, **99**, 1166-1187.

BENIGNO, Gianluca, and THOENISSEN, Christoph (2003), "Equilibrium Exchange Rates and UK Supply Side Performance", *Economic Journal,* **113**, 103-24.

BENIGNO, Pierpaolo (2004), "Optimal Monetary Policy in a Currency Area", *Journal of International Economics,* **63**(2), 293-320.

BERTOLA, Giuseppe (1990), "Job Security, Employment and Wages", *European Economic Review,* **34**(4), 851-879.

BETTS, Caroline and DEVEREUX, Michael (1996), "The Exchange Rate in a Model with Pricing-to-Market", *European Economic Review,* **40**, 1007-1021.

BETTS, Caroline and DEVEREUX, Michael (2000), "Exchange Rate Dynamics in a Model of Pricing to Market", *Journal of International Economics,* **50**, 215-244.

BETTS, Caroline and DEVEREUX, Michael (2001), "The International Effects of Monetary and Fiscal Policy in a Two-Country Model", in G. Calvo, R. Dornbusch and M. Obstfeld (eds.), *Money, Capital Mobility and Trade: Essays in Honor of Robert Mundell,* pp. 9-52 (Cambridge, Mass.: MIT Press).

BLADES, Derek, and ROBERTS, David (2002), "Measuring the non-observed economy", Statistics Brief 5 (Paris: OECD).

BORONDO, Carlos (2000), "International Transmission of Monetary Shocks: MFD vs OR" (mimeo University of Valladolid).

BORONDO, Carlos (2002), "International Transmission of Monetary and Productivity Shocks with Interest Rate Rules" (mimeo University of Valladolid).

BROCK, William A., and MIRMAN, Leonard J. (1972), "Optimal Economic Growth and Uncertainty: The Discounted Case", *Journal of Economic Theory,* **4**, 479-513.

BURDETT, Kenneth, and WRIGHT, Randall (1989), "Unemployment Insurance and Short-Time Compensation: The Effects on Layoffs, Hours per Worker, and Wages", *Journal of Political Economy,* **97**(6), 1479-1496.

BURNSIDE, Craig, and EICHENBAUM, Martin (1996), "Factor-Hoarding and the Propagation of Business-Cycle Shocks", *American Economic Review,* **86**(5), 1154-1174.

BURNSIDE, Craig, EICHENBAUM, Martin, and REBELO, Sergio (1993), "Labor Hoarding and the Business Cycle", *Journal of Political Economy,* **101**(2), 245-273.

CALVO, Guillermo A. (1993), "Staggered Prices in a Utility-Maximising Framework", *Journal of Monetary Economics* **12**(3), 383-398.

CARDIA Emanuela (1991), "The Dynamics of a Small Open Economy in Response to Monetary, Fiscal, and Productivity Shocks", *Journal of Monetary Economics,* **28**, 411-

434.

CAVELAARS, Paul (2001), "Monetary Policy and Competition Policy in a Two-Country, Two-Sector Model" (MEB Series 2001-2, De Nederlandsche Bank).

CENSOLO, Roberto, and ONOFRI, Paolo (1993), "Un'ipotesi di 'Real Business Cycle' per l'economia italiana", *Politica Economica,* **3**, 285-324.

CHARI, V. V., KEHOE, Patrick J., and MCGRATTAN Ellen R., (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", *Econometrica,* **68** (5), 1151-1180.

CHARI, V. V., KEHOE, Patrick J., and MCGRATTAN, Ellen R. (2002), "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?", *Review of Economic Studies,* **69**(3), 533-63.

CHRISTIANO, Lawrence J., EICHENBAUM, Martin, EVANS, Charles, (1992), "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations", *American Economic Review,* **82**(3), 430-450.

CHRISTIANO, Lawrence J., EICHENBAUM, Martin, EVANS, Charles, (1997), "Sticky Price and Limited Participation Models: A Comparison", *European Economic Review,* **41**(6), 1201-1249.

CHRISTODOULAKIS N., S.P. DIMELIS and T. KOLLINTZAS (1994), "Comparison of Business Cycles in the EC: Idiosyncracies and Regularities", *Economica,* **62**, 1-27.

CIMADOMO, Jacopo (2003), "The Effects of Systematic Monetary Policy on Sectors: A Factor Model Analysis" (mimeo.Université Libre de Bruxelles).

CLARIDA, Richard, GALÍ, Jordi, and GERTLER, Mark (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature,* **37**(4), 1661-1707.

COGLEY, Timothy, and NASON, James M. (1995), "Output Dynamics in Real-Business-Cycle Models", *American Economic Review* **85** (3), 492-511.

CONESA, Juan Carlos, DÍAZ-MORENO, Carlos, and GALDÓN-SÁNCHEZ, José Enrique (2002), "Explaining Cross-Country Differences in Participation Rates and Aggregate Fluctuations", *Journal of Economic Dynamics and Control,* **26**(2) pp. 333-345.

COOLEY, Thomas F., and HANSEN, Gary D. (1995), "Money and the Business Cycle", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research* (Princeton: Princeton University Press).

COOLEY, Thomas F., and PRESCOTT, Edward C. (1995), "Economic Growth and

Business Cycles", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research* (Princeton: Princeton University Press).

CORREIA Isabel, NEVES, João C., and REBELO, Sergio (1995), "Business Cycles in a Small Open Economy", *European Economic Review* **39**, pages 1089-1113.

CORSETTI, Giancarlo, and PESENTI, Pierpaolo (2001), "Welfare and Macroeconomic Interdependence", *Quarterly Journal of Economics*, **116**, 421-446.

DANTHINE, Jean-Pierre, and DONALDSON, John B. (1995) "Computing Equilibria of Nonoptimal Economies", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research* (Princeton: Princeton University Press).

DEDOLA, Luca and LIPPI, Francesco (2005), "The Monetary Transmission Mechanism: Evidence from the Industries of Five OECD Countries", *European Economic Review*, **49**(6), 1543-1569.

DORNBUSCH, Rudiger (1976), "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, **84**, 1161-1176.

EUROPEAN COMMISSION (1991), *European Economy*, March (Brussels: European Commission).

EUROPEAN COMMISSION (1995), *European Economy*, No. 3 (Brussels: European Commission).

FLEMING, J. Marcus (1962), "Domestic Financial Policies Under Fixed and Under Floating Exchange Rates", *International Monetary Fund Staff Papers*, **12**, 369-380.

GALÍ, Jordi, and GERTLER, Mark (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics*, **44**(2), 195-222.

GALÍ, Jordi, (2003), "New Perspectives on Monetary Policy, Inflation, and the Business Cycle", in M. Dewatripont, L. Hansen, and S. Turnovsky (eds.), *Advances in Economic Theory*, pp. 151-197, vol. III (Cambridge: Cambridge University Press).

GANLEY, Joe and SALMON, Chris (1997), "The Industrial Impact of Monetary Policy Shocks: Some Stylised Facts" (Working Paper 68, Bank of England).

GREENWOOD, Jeremy, and HERCOWITZ, Zvi (1991), "The Allocation of Capital and Time over the Business Cycle", *Journal of Political Economy*, **99**, 1188-1215.

GREENWOOD, Jeremy, ROGERSON, Richard, and WRIGHT, Randall (1995), "Household Production in Real Business Cycle Theory", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research* (Princeton: Princeton University Press).

HALL, Robert E. (1997), "Macroeconomic Fluctuations and the Allocation of Time", *Journal of Labor Economics*, **15**, s223-s250.

HANSEN Gary D. (1985), "Indivisible labor and the business cycle", *Journal of Monetary Economics*, **16**, 309-327.

HANSEN Gary D., and WRIGHT, Randall (1992), "The Labor Market in Real Business Cycle Theory", *Federal Reserve Bank of Minneapolis, Quarterly Review*, Spring, 2-12.

HAU, Harald (2000), "Exchange rate determination: the role of factor price rigidities and nontradables", *Journal of International Economics*, **50**, 421-447.

HODRICK, Robert J., and PRESCOTT, Edward C. (1997), "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit, and Banking*, **29**, 1-16.

HOPENHAYN, Hugo, and ROGERSON, Richard (1993), "Job Turnover and Policy Evaluation: A General Equilibrium Analysis", *Journal of Political Economy* **101**, pages 915-938.

HORNSTEIN, Andreas, and PRESCOTT, Edward C. (1993), "The Firm and the Plant in General Equilibrium Theory", in R. Becker, M. Boldrin, R. Jones and W. Thomson (eds.), *General Equilibrium, Growth, and Trade II: The Legacy of Lionel McKenzie*, 393-410 (San Diego: Academic Press).

İMROHOROĞLU, Ayse (1989), "Cost of Business Cycles with Indivisibilities and Liquidity Constraints", *Journal of Political Economy*, **97**(6), 1364-1383.

IOE (1985), *Adapting the Labour Market* (Geneva: International Organization of Employers).

KIM, Soyoung (1999) "International Transmission of the US Monetary Policy Shocks: Evidence from VAR's", *Journal of Monetary Economics*, 48(2), 339-372.

KIMBALL, Miles S. (1995), "The Quantitative Analytics of the Basic Neomonetarist Model", *Journal of Money, Credit and Banking*, **27**, 1241-1277.

KING, Robert G., PLOSSER, Charles I., and REBELO, Sergio T. (1988), "Production Growth and Business Cycle: the Basic Neoclassical Model", *Journal of Monetary Economics*, **21**, 195-232.

KYDLAND, Finn E., and PRESCOTT, Edward C. (1977), "Rules Rather Than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, **85** (3), 473-91.

KYDLAND, Finn E., and PRESCOTT, Edward C. (1982), "Time to Build and Aggregate Fluctuations", *Econometrica*, **50**(6), 1345-1370.

KYDLAND, Finn E., and PRESCOTT, Edward C. (1990), "Business Cycles: Real

Facts and a Monetary Myth", *Federal Reserve Bank of Minneapolis Quarterly Review* **14** (2), Spring, 3-18.

KYDLAND, Finn E., and PRESCOTT, Edward C. (1991), "Hours and Employment Variation in Business Cycle Theory", *Economic Theory* **1**, 63-81.

KRUSELL, Per, and SMITH, Anthony A., Jr. (2003), "Consumption–Savings Decisions with Quasi–Geometric Discounting", *Econometrica*, **71** (1), 365-375.

LANE, Philip R. (2001), "The New Open Economy Macroeconomics: A Survey", *Journal of International Economics*, **54**, 235-266.

LAZEAR, Edward P. (1990), "Job Security Provisions and Employment", *The Quarterly Journal of Economics*, **105**(3), 699-726.

LLAUDES, Ricardo (2003), "Monetary Policy Shocks In A Two-Sector Open Economy: An Empirical Study" (mimeo Johns Hopkins University).

LONG, John B., Jr, and PLOSSER, Charles I. (1983), "Real Business Cycles", *Journal of Political Economy*, **91**, 39-69.

LUCAS, Robert E., Jr., and RAPPING, Leonard (1969), "Real Wages, Employment and Inflation", *Journal of Political Economy*, **77**, 721-754.

LUCAS, Robert E., Jr. (1972), "Expectations and the Neutrality of Money", *Journal of Economic Theory*, **4**, 103-124.

LUCAS, Robert E., Jr. (1974), "Equilibrium Search and Unemployment", *Journal of Economic Theory*, **7**, 188-209.

LUCAS, Robert E., Jr. (1976), "Econometric Policy Evaluation: A Critique", *Carnegie-Rochester Series on Public Policy*, **1**, 19-46.

LUCAS, Robert E., Jr. (1977), "Understanding Business Cycles", *Carnegie-Rochester Series on Public Policy*, **5**, 7-29.

LUCAS, Robert E., Jr. (1978), "Asset Prices in an Exchange Economy", *Econometrica*, **46**(6), 1429-45.

LUCAS, Robert E., Jr. (1979), "An Equilibrium Model of the Business Cycle", *Journal of Political Economy*, **83**, 1,113-1,144.

LUCAS, Robert E., Jr. (1981), *Studies in Business-Cycle Theory*, Introduction (Cambridge, Mass.: MIT Press).

LUCAS, Robert E., Jr. (1987), *Models of Business Cycles* (Oxford: Basil Blackwell).

LUCAS, Robert E., Jr. (1988), "Money Demand in the United States: A Quantitative Review", *Carnegie Rochester Conference Series on Public Policy*, **29**, 137-167.

LUCAS, Robert E., Jr. (1995), *Autobiography* (http://www.nobel.se/economics/laureates/1995/lucas-autobio.html).

MACURDY, Thomas E. (1981), "An Empirical Model of Labor Supply in a Life-Cycle Setting", *Journal of Political Economy*, **89**(6), 1059-1085.

MAFFEZZOLI, Marco (2001), "Non-Walrasian Labour Markets and Real Business Cycles", *Review of Economic Dynamics*, **4**(4), 860-892.

MARCHETTI, Domenico J., and NUCCI, Francesco (2001), "Labor effort over the business cycle" (Temi di discussione del Servizio Studi 424, Banca d'Italia).

MARK, Nelson (2001), *International Macroeconomics and Finance* (Oxford: Blackwell Publishers).

MENDOZA E. (1991), "Real Business Cycles in a Small Open Economy", *American Economic Review* **81**, 797-818.

MCCALLUM, Bennet T., and Edward NELSON (1999), "Nominal Income Targeting in an Open-Economy Optimizing Model", *Journal of Monetary Economics*, **43**, 553-578.

MUNDELL, Robert A. (1962), "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability", *International Monetary Fund Staff Papers*, **12**, 70-79.

MUNDELL, Robert A. (1963), "Capital Mobility and Stabilization Policy Under Fixed and Flexible Exchange Rates", *Canadian Journal of Economics and Political Science*, **29**, 475-485.

MUTH, John F. (1961), "Rational Expectations and the Theory of Price Movements", *Econometrica*, **29**, 315-335.

OBSTFELD, Maurice, and ROGOFF, Kenneth (1995), "Exchange Rate Dynamics Redux", *Journal of Political Economy*, **103**, 624-660.

OBSTFELD, Maurice, and ROGOFF, Kenneth (1996), *Foundations of International Macroeconomics*, (Cambridge, Mass.: MIT Press).

OBSTFELD, Maurice, and ROGOFF, Kenneth (2000), "New Directions for Stochastic Open Economy Models", *Journal of International Economics*, **51**, 117-153.

OECD (1994), *The OECD Jobs Study, Evidence and Explanations*, Part II (Paris: OECD).

OECD (1999), *The Employment Outlook* (Paris: OECD).

OSUNA, Victoria, and RÍOS-RULL, Jose-Victor (2003), "Implementing the 35 Hour Workweek by Means of Overtime Taxation", *Review of Economic Dynamics*, **6**(1), pages 179-206.

PAPPA, Evi (2004), "Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World", *Journal of Monetary Economics*, **16**(4), 753-779.

PEERSMAN, Gert and SMETS, Frank (2005), "The Industry Effect of Monetary Policy in the Euro Area", *Economic Journal*, **115**, 319-342.

PRESCOTT, Edward C. (2002), "Non-Convexities in Quantitative General Equilibrium Studies of Business Cycles" (Staff Report 312, Federal Reserve Bank of Minneapolis).

PRESCOTT, Edward C., and TOWNSEND, Robert M. (1984), "Pareto optima and competitive equilibrium with adverse selection and moral hazard", *Econometrica*, **52**, 21–45.

RAVN, Morten O. (1997), "International Business Cycles in Theory and in Practice", *Journal of International Money and Finance*, **16**(2), 255-283.

ROGERSON, Richard (1988), "Indivisible labor, lotteries, and equilibrium", *Journal of Monetary Economics*, **21**, 3-16.

ROGERSON, Richard (1988), "Recursive Competitive Equilibrium in Multi-Sector Economies", *International Economic Review*, **29**(3), 419-430.

ROTEMBERG, Julio J. (1982), "Sticky prices in the United States", *Journal of Political Economy*, **90**, 1187-1211.

ROTEMBERG, Julio J., and WOODFORD, Michael (1995), "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets", in T.F. Cooley (ed.), *Frontiers of Business Cycle Research*, pp. 243-293 (Princeton: Princeton University Press).

SAMUELSON, Paul A. (1958), "An Exact Consumption-Loan Model of Interest with or without the Contrivance of Money", *Journal of Political Economy*, **66**.

SARNO, Lucio (2001), "Towards a New Paradigm in Open Economy Modeling: Where Do We Stand?", *Federal reserve Bank of St. Louis Review*, **83**(3), 21-36.

SCHIVARDI, Fabiano, and TORRINI, Roberto (2004), "Firm size distribution and employment protection legislation in Italy", (Temi di discussione del Servizio Studi 504, Banca d'Italia).

SCHLITZER, Giuseppe (1995), "Business Cycles in Italy: A Statistical Investigation", *European Journal of Political Economy*, **11**, 683-689.

SCHMITT-GROHE, Stephanie, and URIBE, Martin (2003), "Closing Small Open Economy Models", *Journal of International Economics*, **61**(1), 163-185.

SCHNEIDER, Friedrich and ENSTE, Dominik H. (2000) "Shadow economies around the world: size, causes and consequences" (Working Paper 26, International Monetary Fund).

SCHNEIDER, Friedrich and ENSTE, Dominik H. (2002) "Hiding in the shadows. The growth of the underground economy" (Economic Issues 30, International Monetary Fund).

STOCK James H., and WATSON, Mark W. (1999), "Business Cycle Fluctuations in US Macroeconomic Time Series", in J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, Vol 1A. (Amsterdam: Elsevier/North-Holland).

TILLE, Cédric (2001), "The Role of Consumption Substitutability in the International Transmission of Shocks", *Journal of International Economics*,, **53**, 421-444.

UHLIG, Harald (1996), "A Law of Large Numbers for Large Economies", *Economic Theory*, **8**, 41-50.

UHLIG, Harald (1999), "A Toolkit for Analizing Nonlinear Dynamic Stochastic Models Easily", in R. Marimon and A. Scott (eds.), *Computational Methods for the Study of Dynamic Economies* (Oxford: Oxford University Press).

WARNOCK, Francis E. (2003), "Exchange Rate Dynamics and the Welfare Effects of Monetary Policy in a Two-Country Model with Home-Product Bias", *Journal of International Money and Finance*, **22**, 343-363.

ZIZZA, Roberta (2002), "Metodologie di stima dell'economia sommersa: un'applicazione al caso italiano", (Temi di discussione del Servizio Studi 463, Banca d'Italia).

List of Figures

33. Responses of Home real output to one percent shocks, for $\theta = 2$.

34. Responses of Home employment (total) to one percent shocks, for $\alpha = 1$ and $\alpha < 1$

Figure 1: Closed Economy Model, $\theta = 1$, $\gamma = 0$.

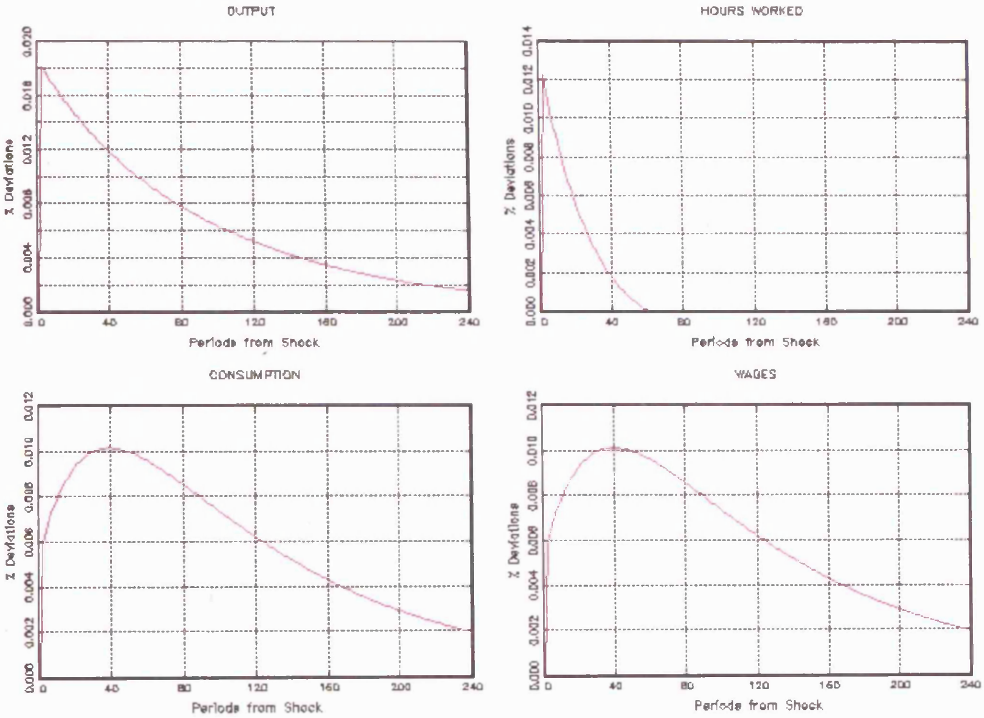


Figure 2: Closed Economy Model, $\theta = 1$, $\gamma = 0.5$.

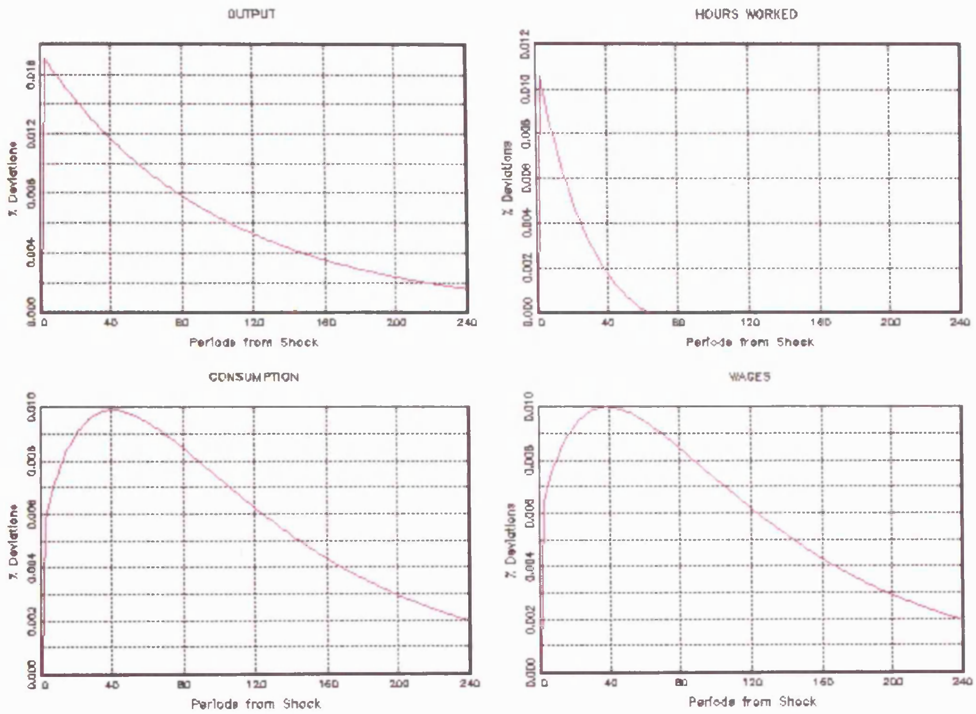


Figure 3: Closed Economy Model, $\theta = 1$, $\gamma = 1$.

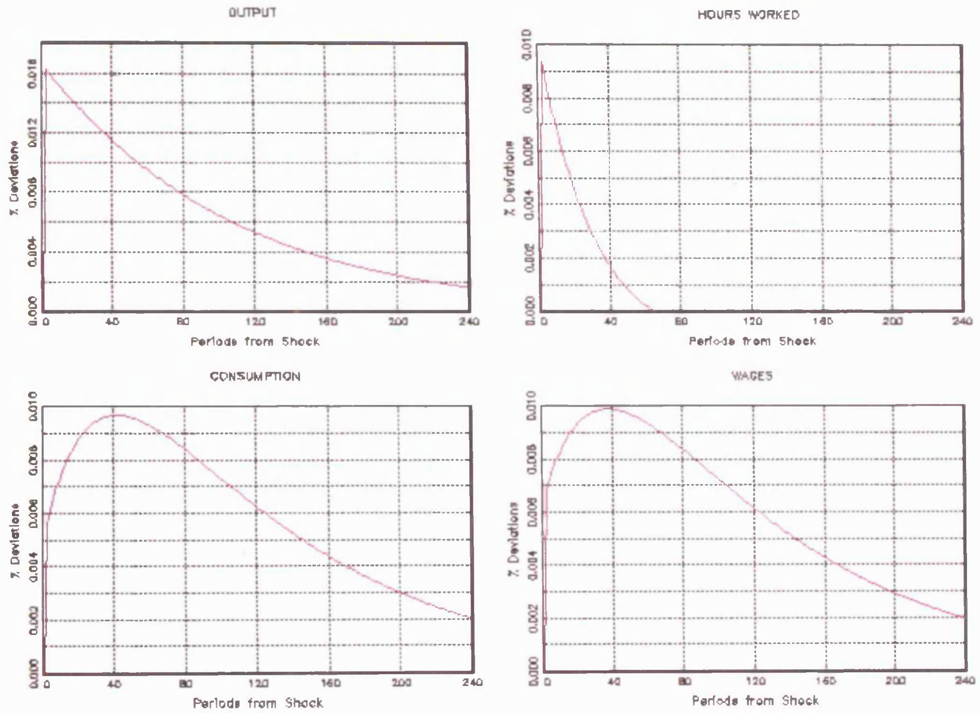


Figure 4: Open Economy Model, $\theta = 1$, $\gamma = 0$: Technology Shock.

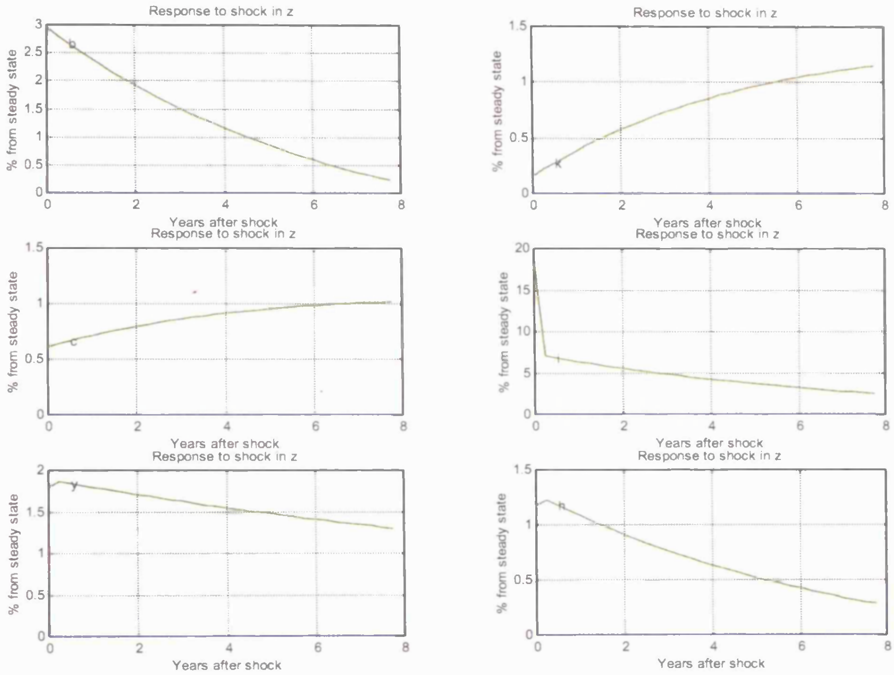


Figure 5: Open Economy Model, $\theta = 1$, $\gamma = 0$: Interest Rate Shock.

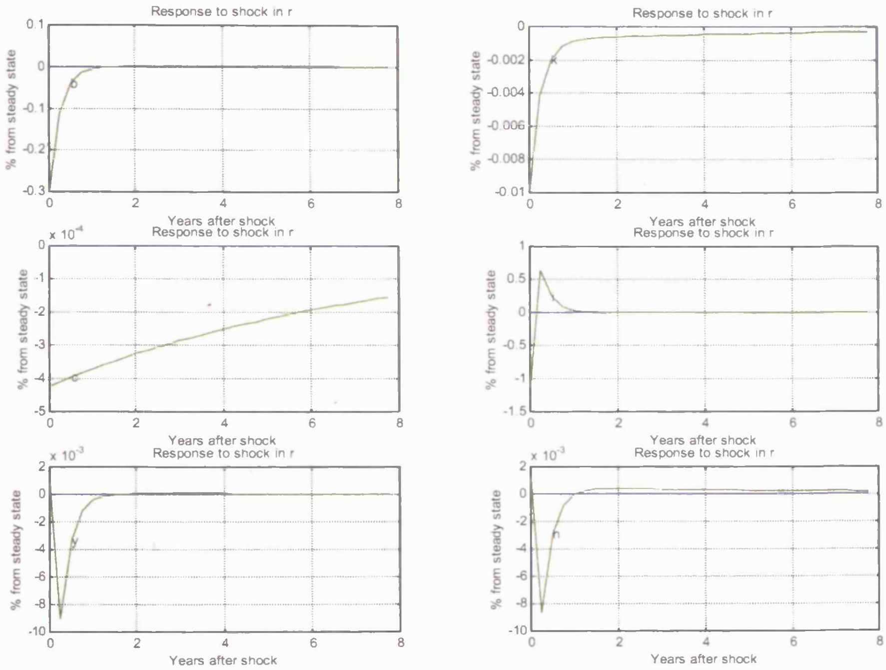


Figure 6: Open Economy Model, $\theta = 1$ **,** $\gamma = 0.5$: Technology Shock.

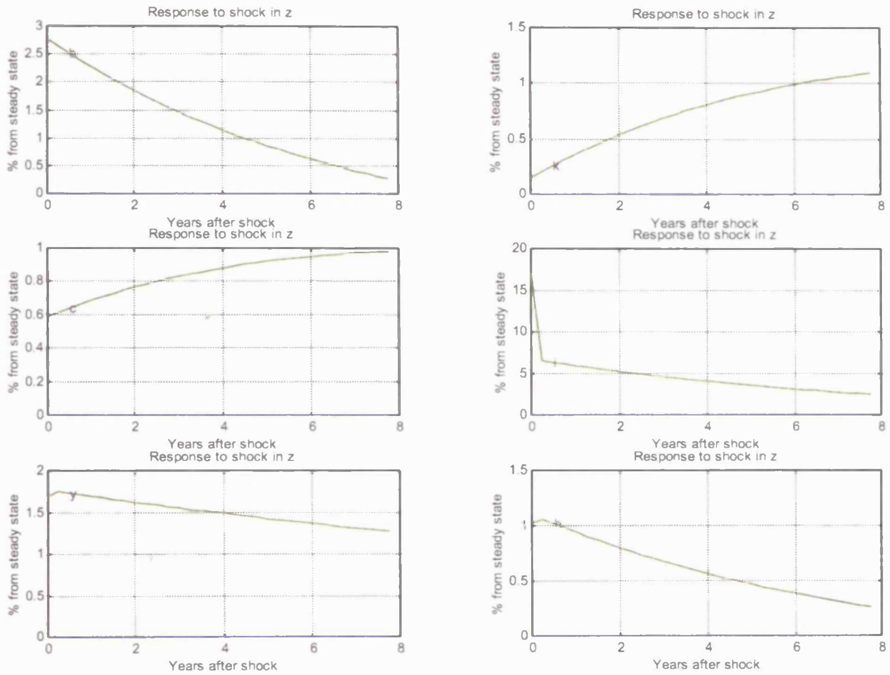


Figure 7: Open Economy Model, $\theta = 1$, $\gamma = 0.5$: Interest Rate Shock.

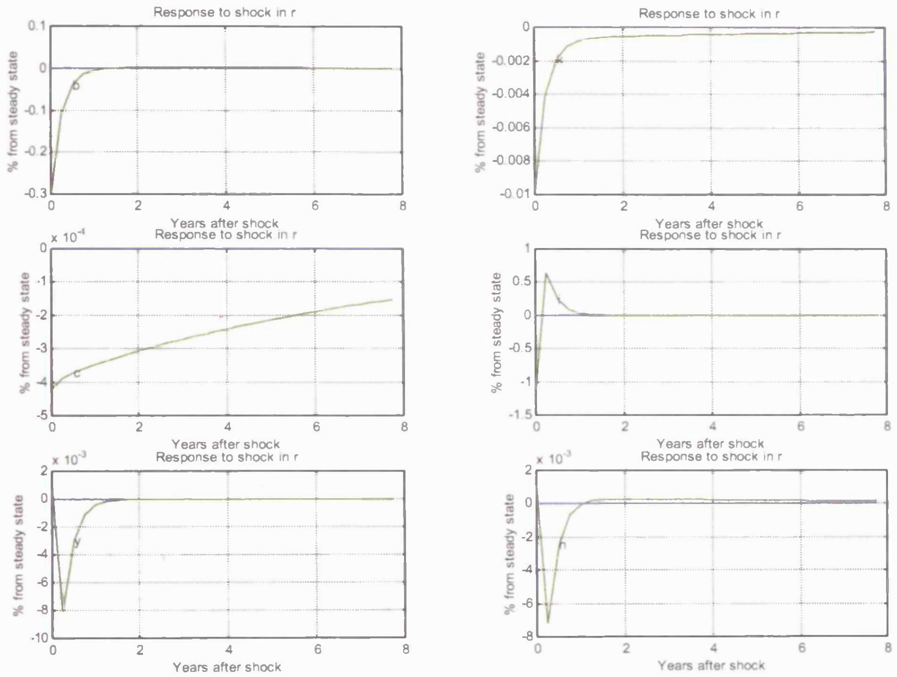


Figure 8: Open Economy Model, $\theta = 1$ **,** $\gamma = 1$: Technology Shock.

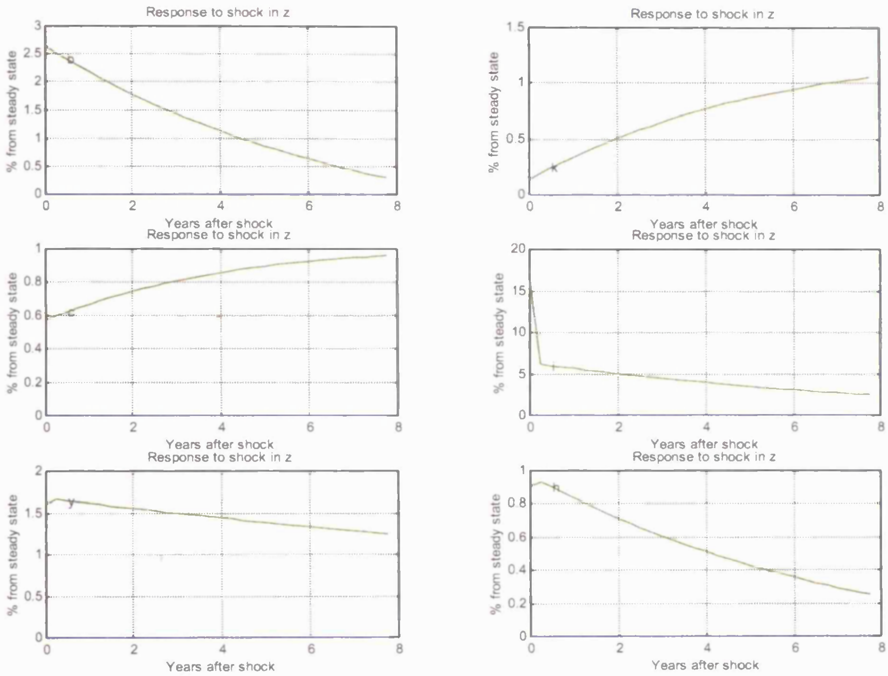


Figure 9: Open Economy Model, $\theta = 1$, $\gamma = 1$: Interest Rate Shock.

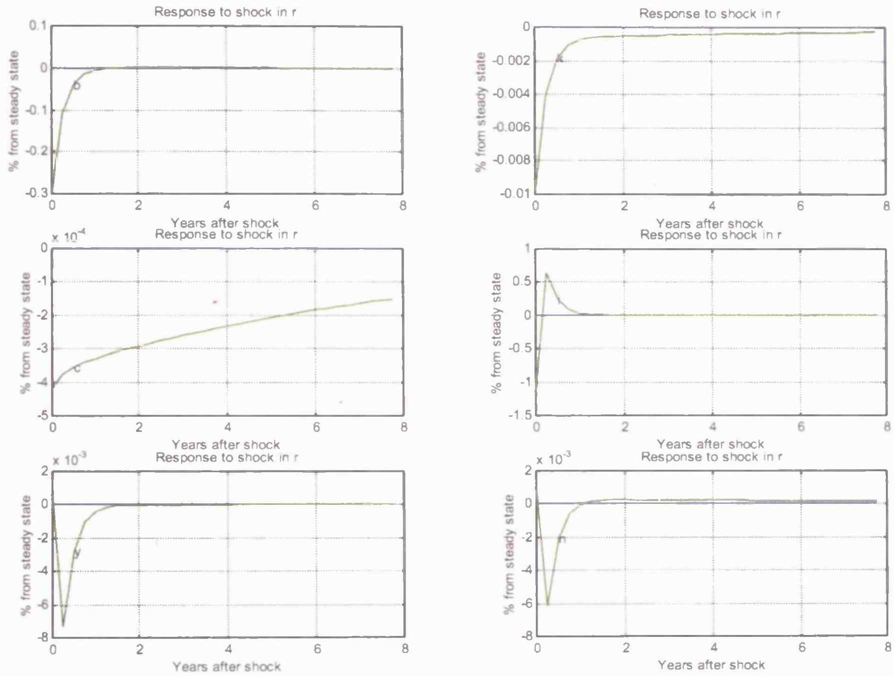


Figure 10: Labour Adjustment Cost Model, $\theta = 2$, $\gamma = 0$, $\phi = 1$.

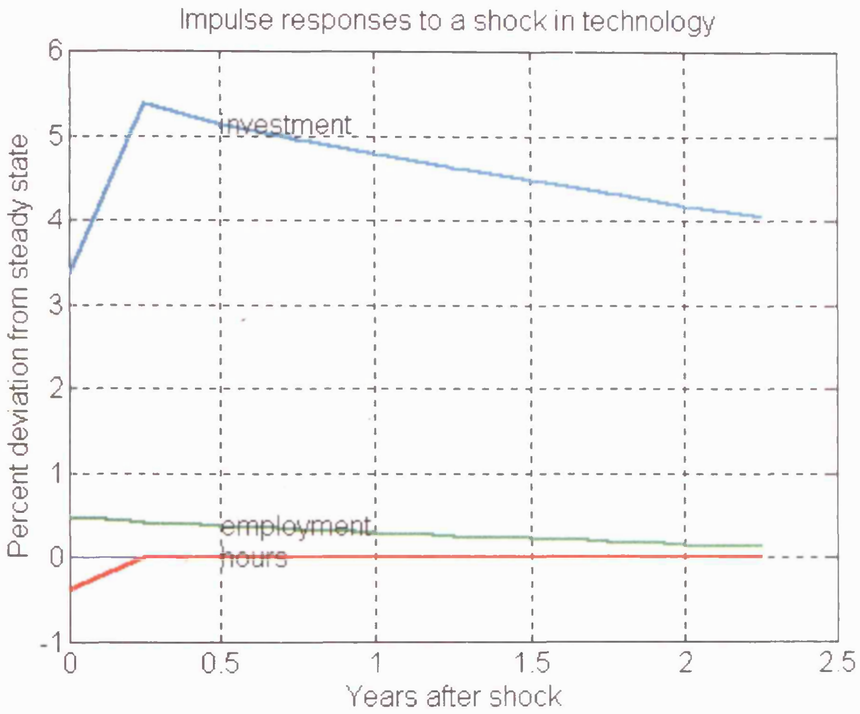


Figure 11: The long-run demand and supply for relative output

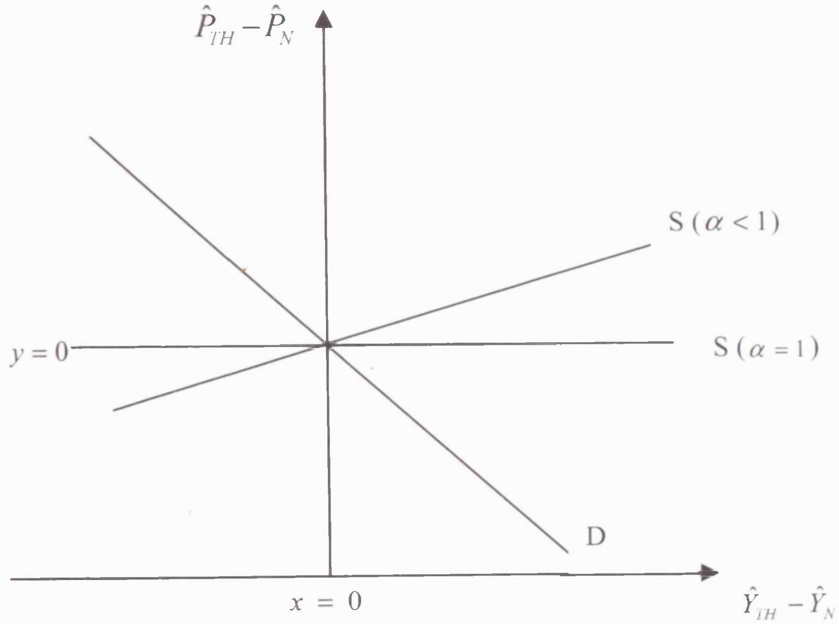
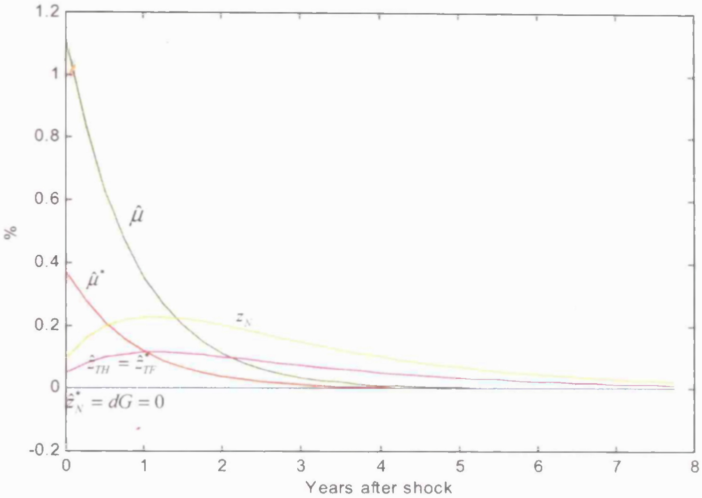
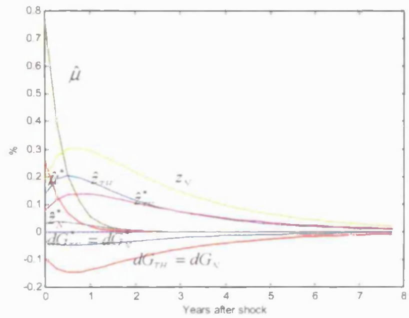


Figure 12: Responses of Home consumption to one percent shocks*



* = Baseline parametrisation.

Figure 13: Responses of Home consumption in the model without the choice of allocations



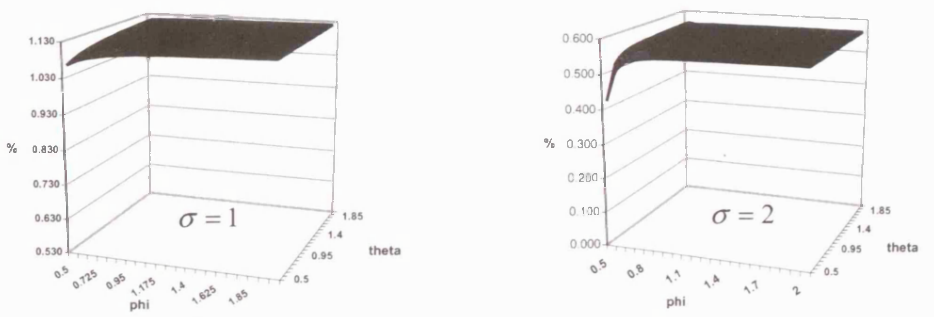
* = Baseline parametrisation.

Chart 1: Responses of Home consumption to one percent government expenditure shocks*

	dG_{TH}	dG_N	dG^*_{TF}	dG^*_N
	$\alpha = 0.85$			
1	-0.003	-0.007	-0.003	0.000
5	-0.009	-0.019	-0.009	0.000
21	-0.004	-0.008	-0.004	0.000
	$\alpha = 0.7$			
1	-0.005	-0.011	-0.005	0.000
5	-0.015	-0.031	-0.015	0.000
21	-0.008	-0.016	-0.008	0.000

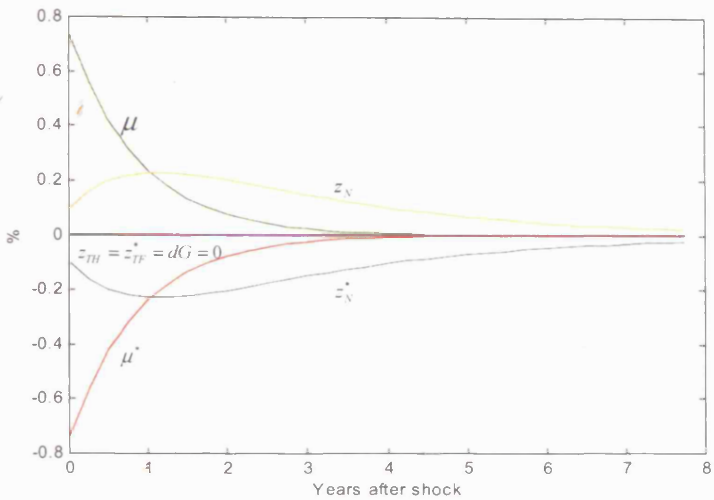
* = All other parameters as in the baseline parametrisation.

Figures 14 and 15: Responses of Home consumption at t=1 after a Home monetary shock, for different values of σ, ϕ and θ.*



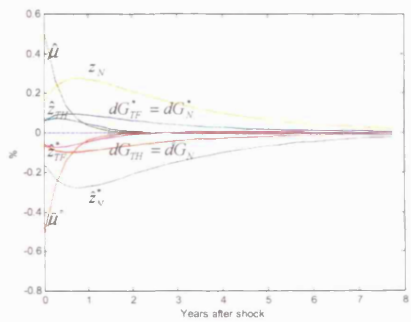
* = All other parameters as in the baseline parametrisation.

Figure 16: Responses of the real exchange rate to one percent shocks*



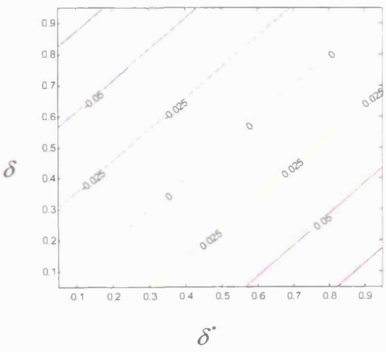
* = Baseline parametrisation.

Figure 17: Responses of the real exchange rate in the model without the choice of allocations



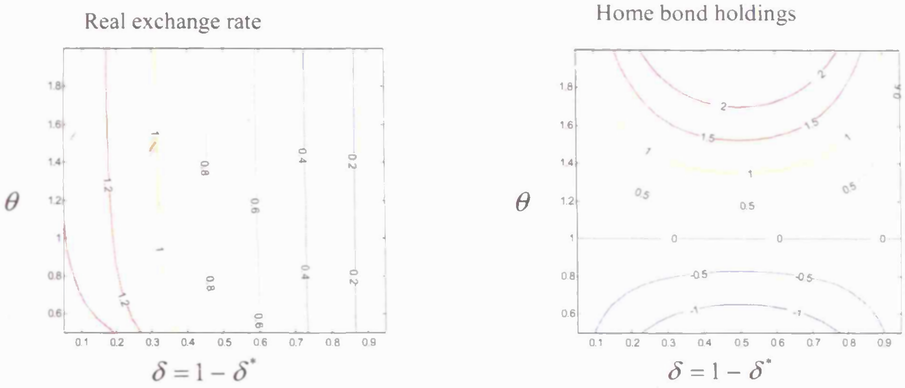
* = Baseline parametrisation.

Figure 18: Response of the real exchange rate at t=1 after a shock in $\hat{z}_{TH}$, for different values of δ and δ^*



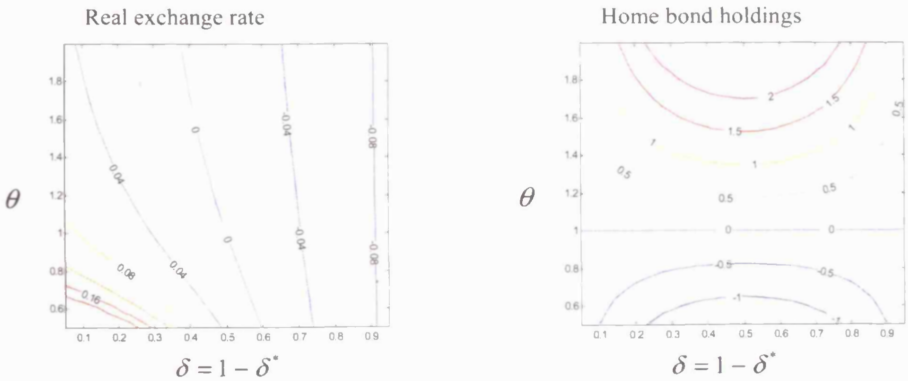
* = All other parameters as in the baseline parametrisation. Response in percentages.

Figures 19 and 20: Response of the real exchange rate and bond holdings at t=1 after a shock in μ, for different values of $\delta = 1 - \delta^*$ and θ *



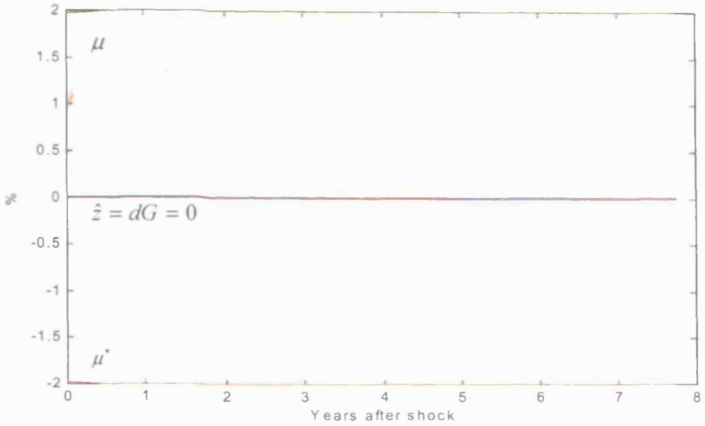
* = All other parameters as in the baseline parametrisation. Response in percentages.

Figures 21 and 22: Response of the real exchange rate and bond holdings at t=1 after a shock in $\hat{z}_{TH}$, for different values of $\delta = 1 - \delta^*$ and θ *



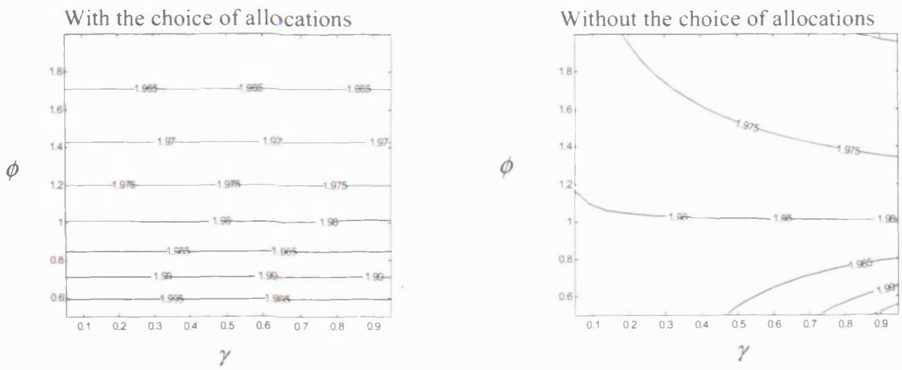
* = All other parameters as in the baseline parametrisation. Response in percentages.

Figure 23: Responses of the nominal exchange rate to one percent shocks*



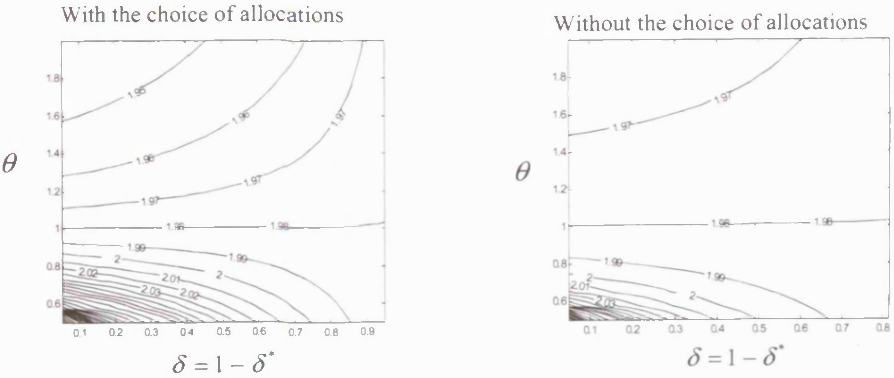
* = Baseline parametrisation.

Figures 24 and 25: Response of the nominal exchange rate at t=1 after a shock in μ, for different values of γ and ϕ *



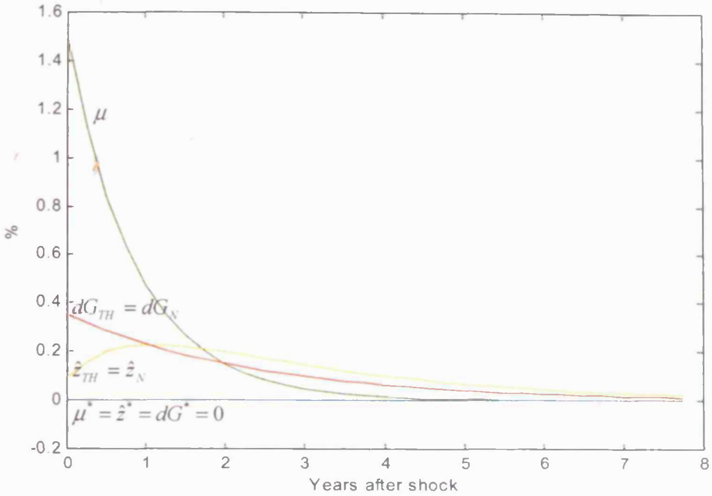
* = All other parameters as in the baseline parametrisation. Response in percentages.

Figures 26 and 27: Response of the nominal exchange rate at t=1 after a shock in μ, for different values of $\delta = 1 - \delta^*$ and θ *



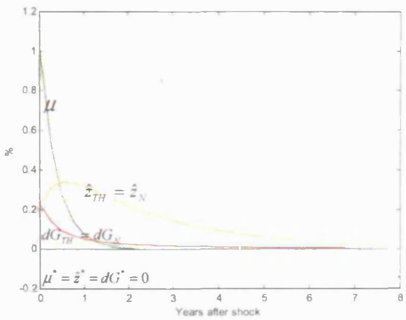
* = All other parameters as in the baseline parametrisation. Response in percentages.

Figure 28: Responses of Home real output to one percent shocks*



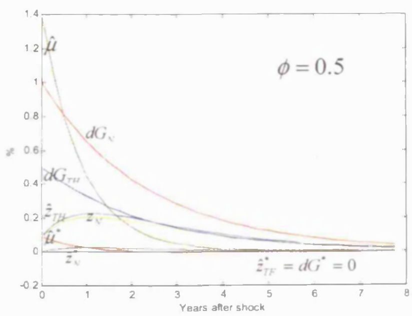
* = Baseline parametrisation. Real output is the sum of tradeable and nontradeable output at constant prices.

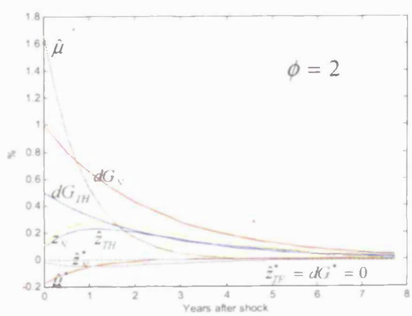
Figure 29: Responses of Home real output in the model without the choice of allocations*



* = See Figure 18.

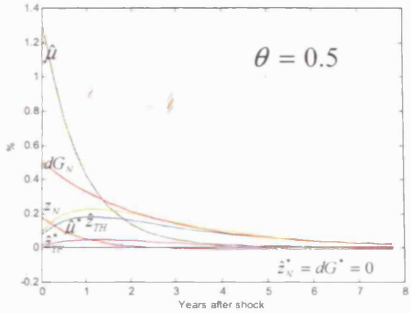
Figures 30 and 31: Responses of Home real output to one percent shocks, for different values of ϕ*

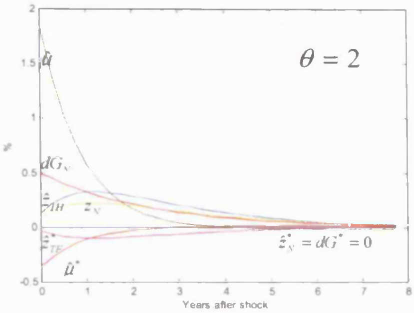




* = Baseline parametrisation

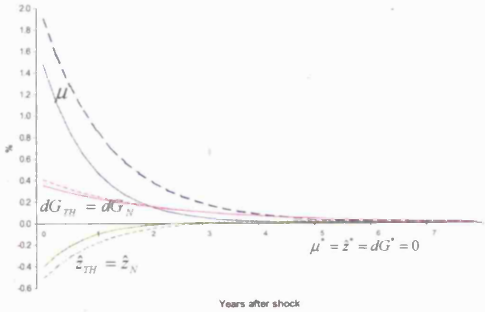
Figures 32 and 33: Responses of Home real output to one percent shocks, for different values of θ *





* = Baseline parametrisation

Figure 34: Responses of Home employment (total) to one percent shocks, for $\alpha = 1$ and $\alpha < 1$ *



* = Baseline parametrisation. Solid lines represent $\alpha = 1$, broken lines represent $\alpha = 0.85$.